Lecture 10 - Class Activity: Multiple Regression

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Analysis of Net Primary Production in Forests: A Modern Tidyverse Approach

Based on Michaletz et al. (2014) data

Introduction

This analysis examines the relationships between Net Primary Production (npp) and various climate and forest characteristics across global forest sites. We'll explore multicollinearity, model selection, and variable transformations.

Key Learning Objectives:

- Understand multicollinearity in multiple regression
- Learn model diagnostics and assumption checking
- Practice variable selection techniques
- Apply data transformations appropriately

Load Required Packages

```
# Load required packages
library(tidyverse)    # For data manipulation and visualization
library(car)          # For regression diagnostics (VIF, etc.)
library(corrplot)     # For correlation plots
library(GGally)       # For pairs plots
library(broom)        # For tidy model outputs
library(performance)  # For model performance metrics
library(see)          # For better diagnostic plots
```

Load and Explore the Data

```
# Load the forest npp data
forest_df <- read_csv("data/michaletz_etal_2014_clean.csv")

# Display top lines
head(forest_df)
```

```
# A tibble: 6 × 8
    npp   age biomass season  temp precip   teb leaf
  <dbl> <dbl>   <dbl>  <dbl> <dbl>  <dbl> <dbl> <chr>
1  2084  104.  18198.     11 25.3    1888   0.55 broadleaf
2  2234  333.  54523.     12 26.9    2348.  0.6  broadleaf
3  2714  213.  41358.     12 26.9    2348.  0.6  broadleaf
4  2828  114.  31557.     12 26.7    2784.  1.25 broadleaf
5  2882  113.  21417      12 26.7    2784.  1.25 broadleaf
6   774   79   11188       3 -3.53    408.  1.8  broadleaf
```

Data Preparation

Following the original analysis, we'll focus on the key variables from a cleaned dataframe

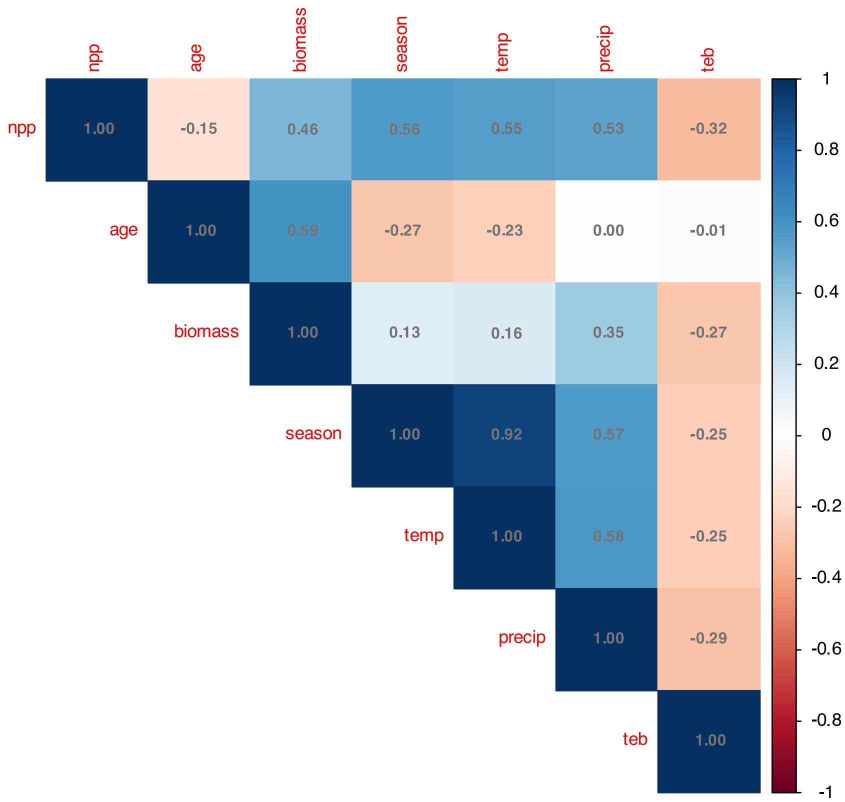
1. Initial Exploration: Variable Relationships and Multicollinearity

Correlation Matrix and Visualization

```r
# Create correlation matrix for numeric variables only
num_vars <- forest_df %>%
  select_if(is.numeric)

# Calculate correlation matrix
cor_matrix <- cor(num_vars, use = "complete.obs")

# Display correlation matrix
cor_matrix
```

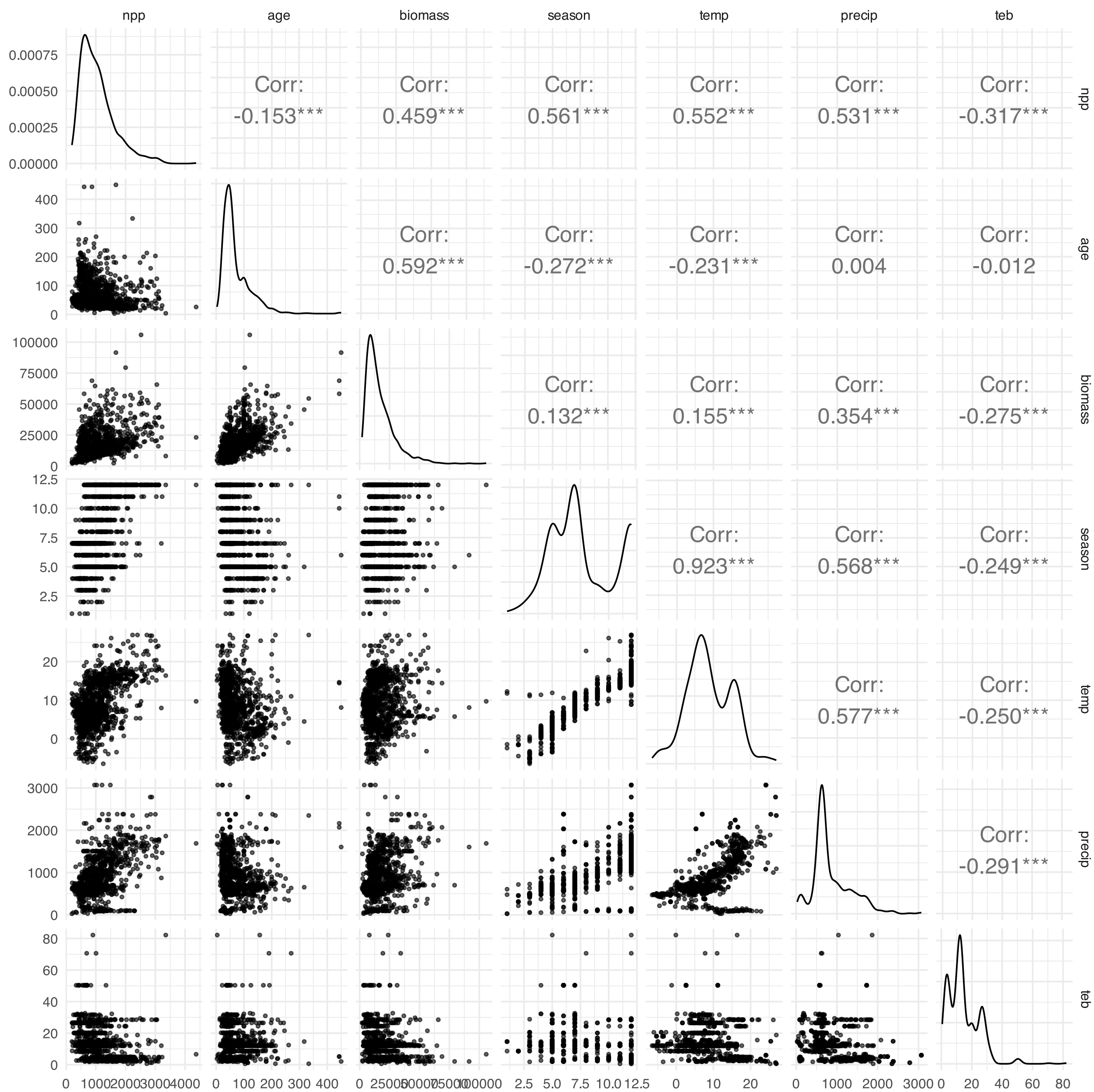
```
               npp          age   biomass     season       temp       precip
npp      1.0000000 -0.153215979  0.4591083  0.5614272  0.5520590  0.531186333
age     -0.1532160  1.000000000  0.5923809 -0.2715014 -0.2307658  0.003642104
biomass  0.4591083  0.592380901  1.0000000  0.1317097  0.1551923  0.353901541
season   0.5614272 -0.271501387  0.1317097  1.0000000  0.9226905  0.567630275
temp     0.5520590 -0.230765849  0.1551923  0.9226905  1.0000000  0.576659967
precip   0.5311863  0.003642104  0.3539015  0.5676303  0.5766600  1.000000000
teb     -0.3167187 -0.011779155 -0.2748724 -0.2491779 -0.2495479 -0.291376661
                teb
npp     -0.31671865
age     -0.01177916
biomass -0.27487245
season  -0.24917787
temp    -0.24954794
precip  -0.29137666
teb      1.00000000
```

```r
# Create a visual correlation plot
corrplot(cor_matrix, method = "color", type = "upper",
         addCoef.col = "grey45", tl.cex = 0.8, number.cex = 0.7)
```



Pairs Plot for Visual Inspection

```
# Create pairs plot to visualize relationships
# This replaces the original pairs() function with ggplot2
forest_df %>%
  select(-leaf) %>%  # Exclude categorical variable for pairs plot
  ggpairs(
    upper = list(continuous = wrap("cor", size = 5)),
    lower = list(continuous = wrap("points", alpha = 0.6, size = 0.8))
  ) +
  theme_minimal()
```



2. Initial Multiple Regression Model

Let's start with a full model including all predictors:

```
# Fit initial model with all predictors (Model 1)
model_init <- lm(npp ~ age + biomass + season + temp +
             precip + teb + leaf, data = forest_df)

# Get model summary
summary(model_init)
```

```

Call:
lm(formula = npp ~ age + biomass + season + temp + precip + teb +
    leaf, data = forest_df)
```

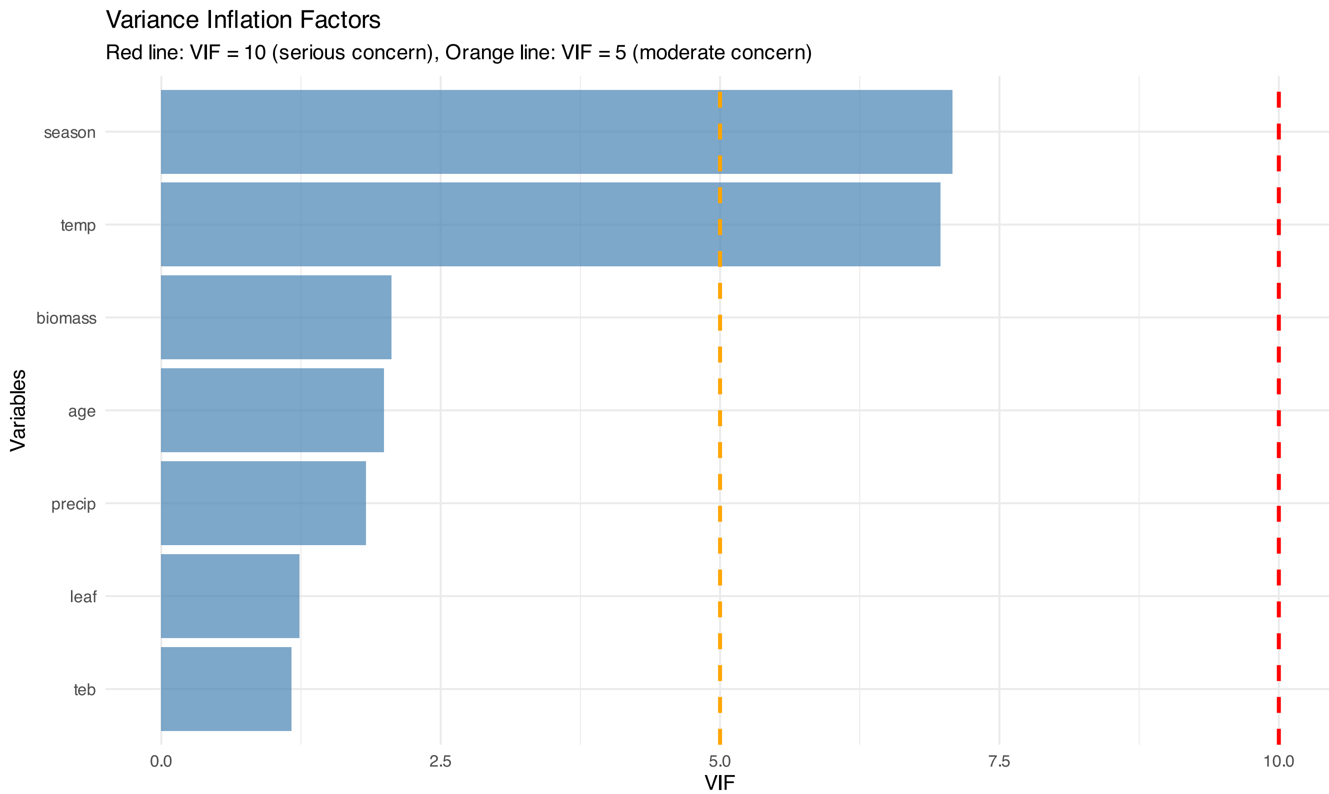
```
Residuals:
     Min       1Q   Median       3Q      Max 
-1331.10  -206.27   -34.09   166.94  2760.41 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  583.92959   53.81169  10.851  < 2e-16 ***
age           -4.78822    0.28518 -16.790  < 2e-16 ***
biomass        0.03154    0.00122  25.848  < 2e-16 ***
season        41.18220    9.49073   4.339 1.55e-05 ***
temp           4.61281    4.37372   1.055 0.291788    
precip         0.09674    0.02852   3.392 0.000716 ***
teb           -2.12880    1.08408  -1.964 0.049794 *  
leafneedle  -267.00569   22.43078 -11.904  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 351.7 on 1212 degrees of freedom
Multiple R-squared:  0.6415,    Adjusted R-squared:  0.6394 
F-statistic: 309.8 on 7 and 1212 DF,  p-value: < 2.2e-16
```

Check for Multicollinearity

```r
# Calculate Variance Inflation Factors (VIF)
vif_values <- vif(model_init)
vif_values
```

```
     age  biomass   season     temp   precip      teb     leaf 
1.993348 2.061766 7.079004 6.972147 1.831127 1.167007 1.236588 
```

```r
# Create a data frame for better visualization
vif_df <- data.frame(
  Variable = names(vif_values),
  VIF = as.numeric(vif_values)
) %>%
  arrange(desc(VIF))

# Visualize VIF values
ggplot(vif_df, aes(x = reorder(Variable, VIF), y = VIF)) +
  geom_col(fill = "steelblue", alpha = 0.7) +
  geom_hline(yintercept = 10, color = "red", linetype = "dashed",
             linewidth = 1) +
  geom_hline(yintercept = 5, color = "orange", linetype = "dashed",
             linewidth = 1) +
  coord_flip() +
  labs(
    title = "Variance Inflation Factors",
    subtitle = "Red line: VIF = 10 (serious concern), Orange line: VIF = 5 (moderate concern)",
    x = "Variables",
    y = "VIF"
  ) +
  theme_minimal()
```



Address Multicollinearity by Removing Growing Season

Based on the original analysis, season and Temperature are highly correlated. Let's remove season:

```r
# Model 2: Remove season due to multicollinearity
model_2 <- lm(npp ~ age + biomass + temp + precip + teb + leaf,
             data = forest_df)

# Model summary
summary(model_2)
```

```

Call:
lm(formula = npp ~ age + biomass + temp + precip + teb + leaf,
    data = forest_df)

Residuals:
     Min       1Q   Median       3Q      Max
-1309.94  -209.40   -42.65   167.71  2898.47

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  7.511e+02  3.785e+01  19.845  < 2e-16 ***
age         -5.004e+00  2.829e-01 -17.690  < 2e-16 ***
biomass      3.180e-02  1.228e-03  25.905  < 2e-16 ***
temp         2.112e+01  2.173e+00   9.718  < 2e-16 ***
precip       1.120e-01  2.851e-02   3.929 9.02e-05 ***
teb         -2.255e+00  1.092e+00  -2.066    0.039 *
leafneedle  -2.634e+02  2.258e+01 -11.666  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 354.3 on 1213 degrees of freedom
Multiple R-squared:  0.6359,    Adjusted R-squared:  0.6341
F-statistic: 353.1 on 6 and 1213 DF,  p-value: < 2.2e-16
```

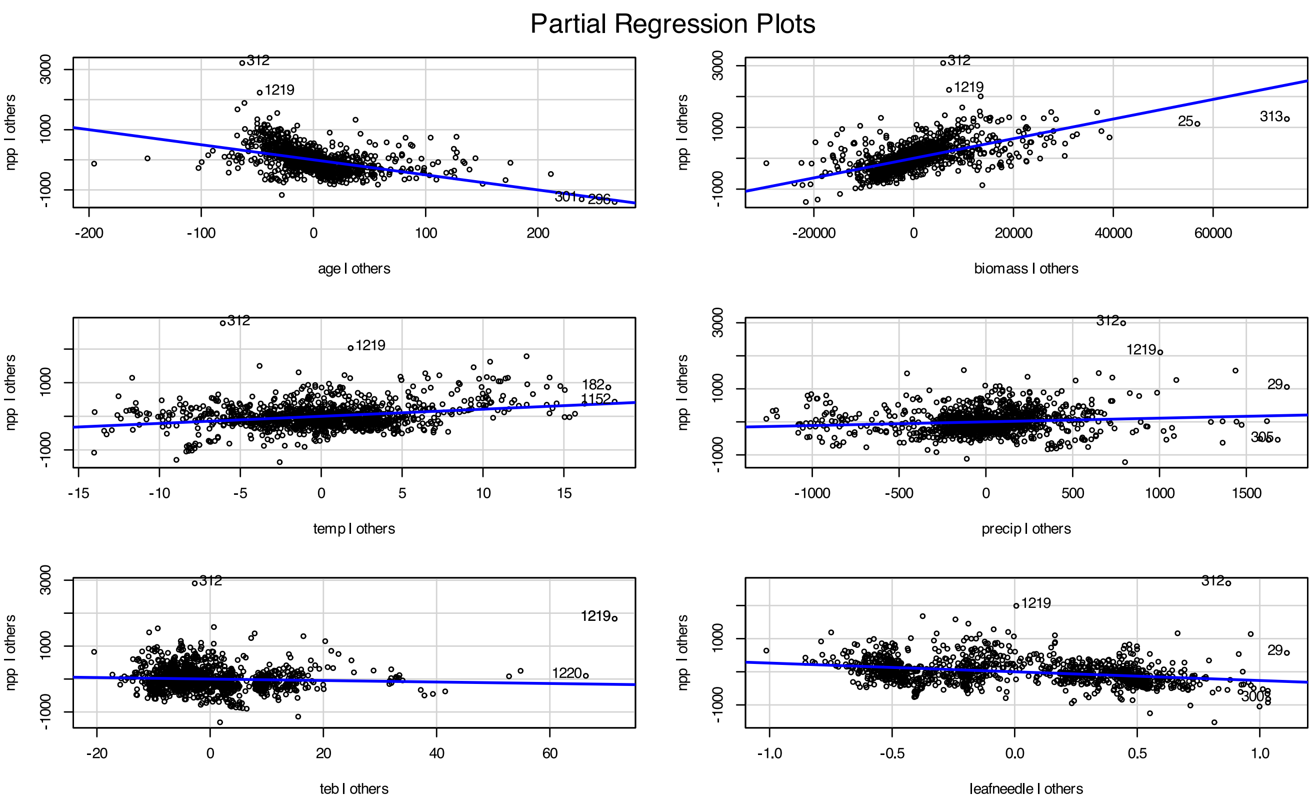
```r
# Check VIF again
vif_values2 <- vif(model_2)
vif_values2
```

```
     age  biomass     temp   precip      teb     leaf
1.932806 2.056724 1.696787 1.803266 1.166162 1.234906
```

3. Exploring Variable Transformations

Check the Shape of Relationships with Partial Regression Plots

```r
# Create partial regression plots (Added Variable Plots)
# This helps us see the relationship between each predictor and response
# after accounting for other variables

par(mfrow = c(2, 3))
avPlots(model_2, main = "Partial Regression Plots")
```




```r
par(mfrow = c(1, 1))
```

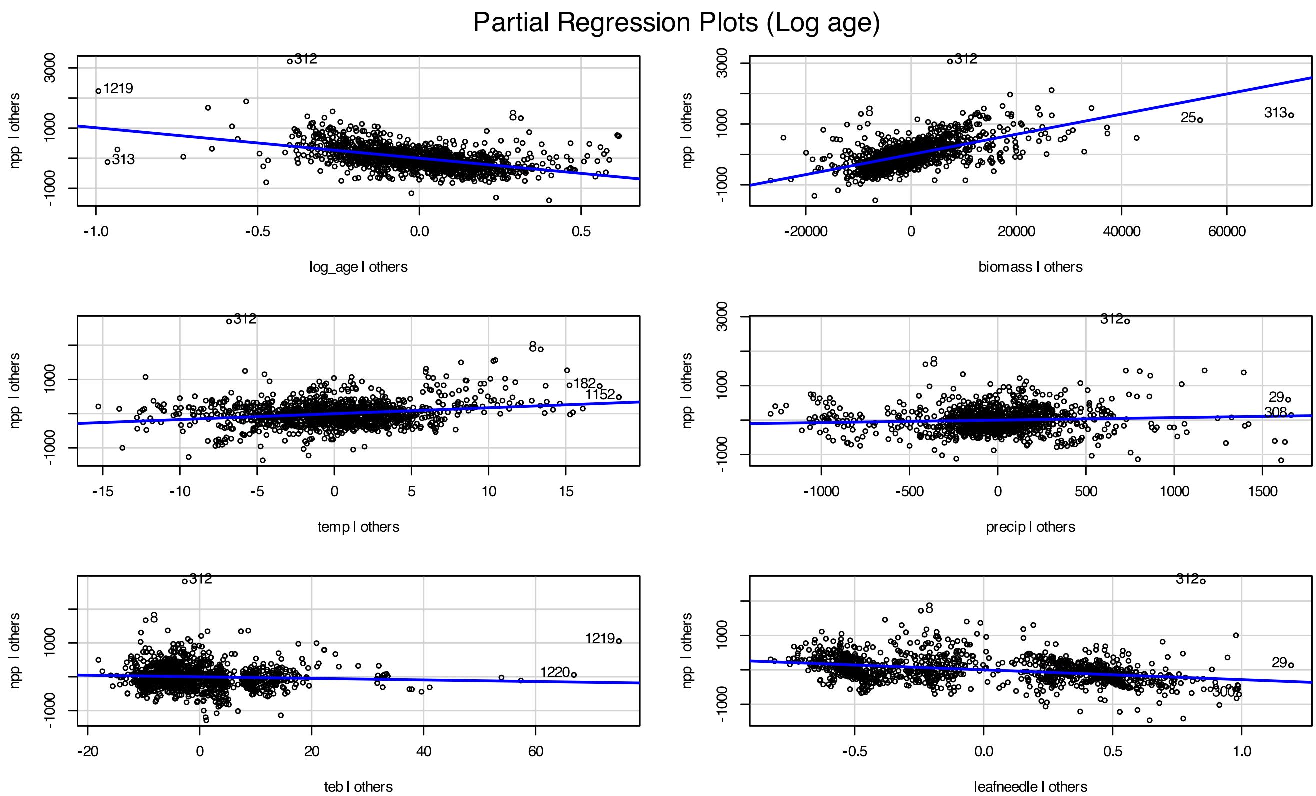
Apply Log Transformation to age

The original analysis found that age showed a curvy relationship. Let's try log transformation: REally what you should do is log transform of the response variable first..

```r
# Create dataset with log-transformed age
forest_df <- forest_df %>%
  mutate(log_age = log10(age))

# Model 3: With log-transformed age
model_3 <- lm(npp ~ log_age + biomass + temp + precip + teb + leaf,
             data = forest_df)


# Model summary
summary(model_3)
```

```

Call:
lm(formula = npp ~ log_age + biomass + temp + precip + teb +
    leaf, data = forest_df)

Residuals:
     Min       1Q   Median       3Q      Max
-1282.97  -203.31   -23.13   163.75  2810.76

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.237e+03  8.961e+01  24.962  < 2e-16 ***
log_age     -1.012e+03  5.064e+01 -19.973  < 2e-16 ***
biomass      3.313e-02  1.194e-03  27.755  < 2e-16 ***
temp         1.723e+01  2.159e+00   7.982 3.33e-15 ***
precip       7.194e-02  2.781e-02   2.587  0.00981 **
teb         -2.315e+00  1.061e+00  -2.183  0.02925 *
leafneedle  -2.852e+02  2.177e+01 -13.105  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 344.7 on 1213 degrees of freedom
Multiple R-squared:  0.6553,    Adjusted R-squared:  0.6536
F-statistic: 384.4 on 6 and 1213 DF,  p-value: < 2.2e-16
```

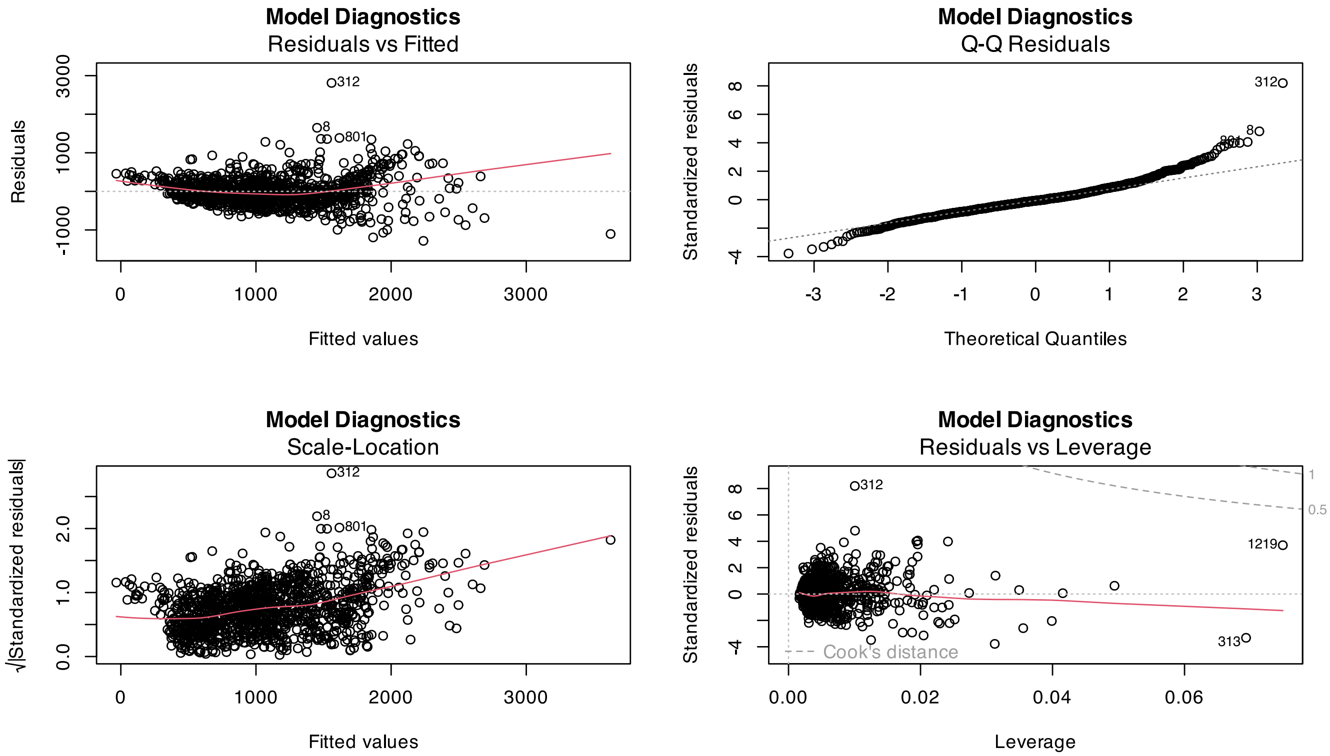
```r
# Compare partial regression plots
par(mfrow = c(2, 3))
avPlots(model_3, main = "Partial Regression Plots (Log age)")
```




```r
par(mfrow = c(1, 1))
```

4. Model Diagnostics and Assumption Checking

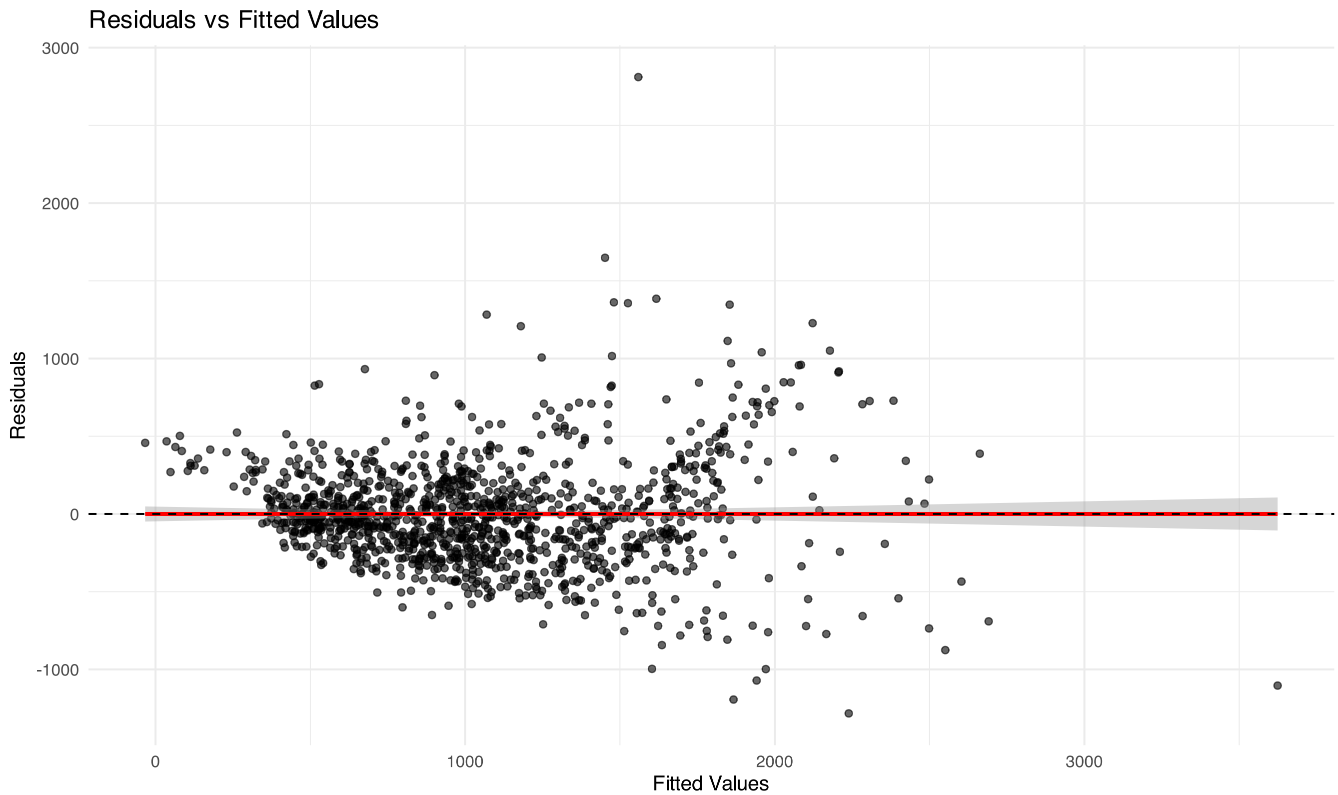
```r
# Create diagnostic plots
par(mfrow = c(2, 2))
plot(model_3, main = "Model Diagnostics")
```




```
par(mfrow = c(1, 1))
```

Residuals - its the upper left above

```
# Check for normality of residuals
residuals_data <- data.frame(
  Fitted = fitted(model_3),
  Residuals = residuals(model_3),
  Standardized_Residuals = rstandard(model_3)
)

# Residuals vs Fitted plot using ggplot
ggplot(residuals_data, aes(x = Fitted, y = Residuals)) +
  geom_point(alpha = 0.6) +
  geom_smooth(method = "lm", color = "red") +
  geom_hline(yintercept = 0, linetype = "dashed") +
  labs(
    title = "Residuals vs Fitted Values",
    x = "Fitted Values",
    y = "Residuals"
  ) +
  theme_minimal()
```



Try Response Variable Transformation

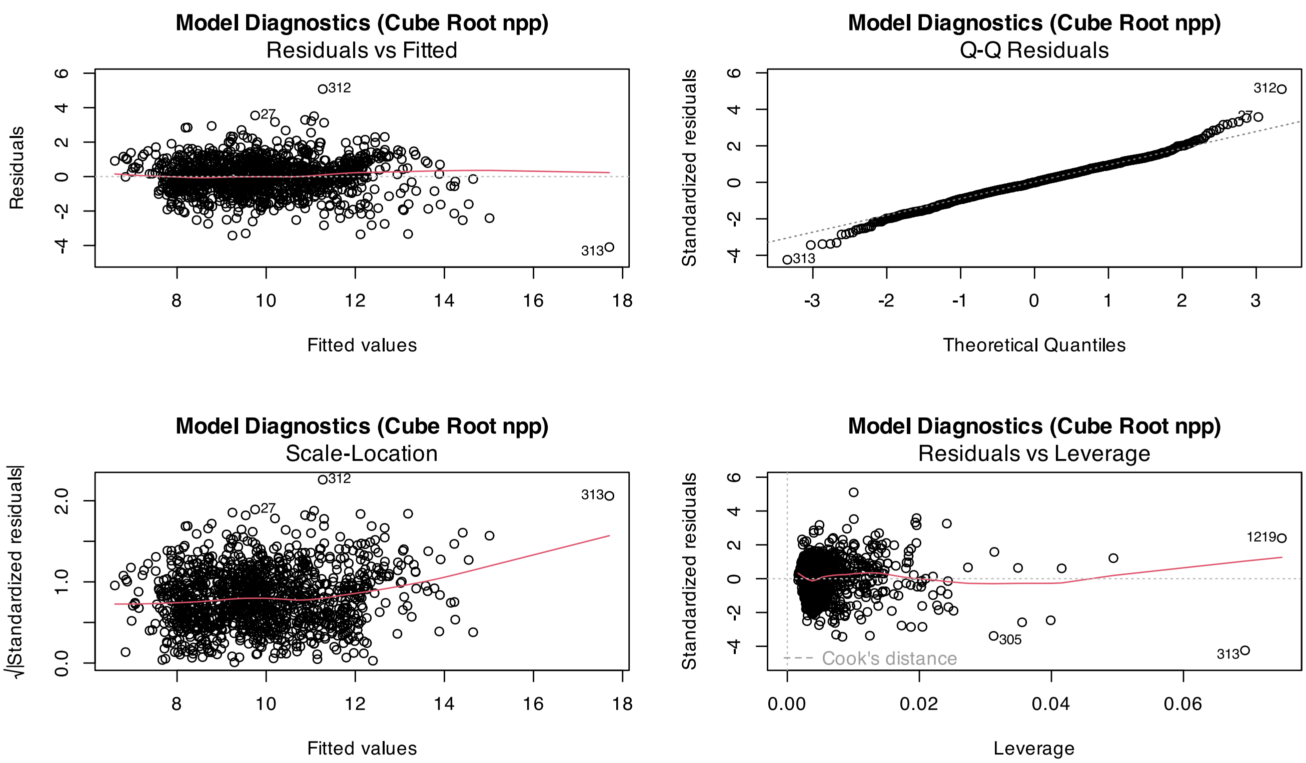
Following the original analysis, let's try a cube root transformation of npp:

```r
# Model 4: Cube root transformation of npp
forest_df <- forest_df %>%
  mutate(npp_cuberoot = npp^(1/3))

model_4 <- lm(npp_cuberoot ~ log_age + biomass + temp + precip +
             teb + leaf, data = forest_df)

summary(model_4)
```

```

Call:
lm(formula = npp_cuberoot ~ log_age + biomass + temp + precip +
    teb + leaf, data = forest_df)

Residuals:
    Min      1Q  Median      3Q     Max
-4.0936 -0.5916 -0.0030  0.6463  5.0795

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.391e+01  2.601e-01  53.489  < 2e-16 ***
log_age     -3.151e+00  1.470e-01 -21.436  < 2e-16 ***
biomass      1.032e-04  3.464e-06  29.791  < 2e-16 ***
temp         4.507e-02  6.265e-03   7.194 1.10e-12 ***
precip       6.361e-05  8.072e-05   0.788    0.431
teb         -1.273e-02  3.078e-03  -4.135 3.79e-05 ***
leafneedle  -1.029e+00  6.317e-02 -16.283  < 2e-16 ***
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1 on 1213 degrees of freedom
Multiple R-squared:  0.6793,	Adjusted R-squared:  0.6777
F-statistic: 428.2 on 6 and 1213 DF,  p-value: < 2.2e-16
```

```r
# Check diagnostics
par(mfrow = c(2, 2))
plot(model_4, main = "Model Diagnostics (Cube Root npp)")
```




```r
par(mfrow = c(1, 1))
```

5. Model Simplification and Comparison

Remove Non-significant Variables

```r
# Model 5: Remove non-significant Precipitation
model_5 <- lm(npp_cuberoot ~ log_age + biomass + temp + teb + leaf,
              data = forest_df)

summary(model_5)
```

```
Call:
lm(formula = npp_cuberoot ~ log_age + biomass + temp + teb +
    leaf, data = forest_df)
```

```
Residuals:
    Min      1Q  Median      3Q     Max 
-4.1085 -0.5919  0.0019  0.6459  5.1261 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.396e+01  2.532e-01  55.120  < 2e-16 ***
log_age     -3.159e+00  1.465e-01 -21.559  < 2e-16 ***
biomass      1.040e-04  3.306e-06  31.465  < 2e-16 ***
temp         4.719e-02  5.658e-03   8.342  < 2e-16 ***
teb         -1.296e-02  3.064e-03  -4.231  2.5e-05 ***
leafneedle  -1.040e+00  6.150e-02 -16.910  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1 on 1214 degrees of freedom
Multiple R-squared:  0.6791,    Adjusted R-squared:  0.6778 
F-statistic: 513.9 on 5 and 1214 DF,  p-value: < 2.2e-16
```

```
# Compare models using AIC
model_comparison <- data.frame(
  Model = c("Model 4 (Full)", "Model 5 (No precip)"),
  AIC = c(AIC(model_4), AIC(model_5)),
  R_squared = c(summary(model_4)$r.squared, summary(model_5)$r.squared),
  Adj_R_squared = c(summary(model_4)$adj.r.squared, summary(model_5)$adj.r.squared)
)

model_comparison
```

```
                Model      AIC R_squared Adj_R_squared
1      Model 4 (Full) 3472.092 0.6792946     0.6777082
2 Model 5 (No precip) 3470.717 0.6791304     0.6778089
```

Model Performance and Interpretation

```
# Get tidy summary of final model
summary(model_5, conf.int = TRUE)
```

```


Call:
lm(formula = npp_cuberoot ~ log_age + biomass + temp + teb + 
    leaf, data = forest_df)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.1085 -0.5919  0.0019  0.6459  5.1261 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.396e+01  2.532e-01  55.120  < 2e-16 ***
log_age     -3.159e+00  1.465e-01 -21.559  < 2e-16 ***
biomass      1.040e-04  3.306e-06  31.465  < 2e-16 ***
temp         4.719e-02  5.658e-03   8.342  < 2e-16 ***
teb         -1.296e-02  3.064e-03  -4.231  2.5e-05 ***
```

```
leafneedle  -1.040e+00  6.150e-02 -16.910  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1 on 1214 degrees of freedom
Multiple R-squared:  0.6791,    Adjusted R-squared:  0.6778
F-statistic: 513.9 on 5 and 1214 DF,  p-value: < 2.2e-16
```

using the sensemaker package

```
library(sensemakr)
# Calculate partial R-squared for all variables at once
partial_r2_sensemakr <- partial_r2(model_5)
partial_r2_sensemakr
```

```
(Intercept)     log_age     biomass        temp         teb  leafneedle
 0.71450239  0.27686109  0.44919401  0.05421045  0.01453129  0.19064332
```

```
# Calculate partial R-squared for each variable
# Using the car package
print("Using car package Anova() with Type III sums of squares:")
```

```
[1] "Using car package Anova() with Type III sums of squares:"
```

```
anova_type2 <- Anova(model_5, type = "III")
print(anova_type2)
```

```
Anova Table (Type III tests)

Response: npp_cuberoot
             Sum Sq   Df  F value    Pr(>F)
(Intercept) 3039.52    1 3038.225 < 2.2e-16 ***
log_age      464.99    1  464.792 < 2.2e-16 ***
biomass      990.47    1  990.043 < 2.2e-16 ***
temp          69.61    1   69.584 < 2.2e-16 ***
teb           17.91    1   17.901 2.502e-05 ***
leaf         286.08    1  285.957 < 2.2e-16 ***
Residuals   1214.52 1214
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Convert F-statistics to partial R-squared
# Partial R² = F * df_num / (F * df_num + df_den)
f_stats <- anova_type2$`F value`[!is.na(anova_type2$`F value`)]
df_num <- anova_type2$Df[!is.na(anova_type2$`F value`)]
df_den <- anova_type2$Df[nrow(anova_type2)]  # Residual df

partial_r2_from_f <- f_stats * df_num / (f_stats * df_num + df_den)

results_table <- data.frame(
  Variable = rownames(anova_type2)[!is.na(anova_type2$`F value`)],
```

```
  F_statistic = f_stats,
  p_value = anova_type2$`Pr(>F)`[!is.na(anova_type2$`F value`)],
  Partial_R_squared = partial_r2_from_f
)

print("Complete results with partial R-squared:")
```

```
[1] "Complete results with partial R-squared:"
```

```
print(results_table)
```

```
     Variable F_statistic       p_value Partial_R_squared
1 (Intercept)  3038.22472  0.000000e+00        0.71450239
2     log_age   464.79225  1.532052e-87        0.27686109
3     biomass   990.04285 2.085787e-159        0.44919401
4        temp    69.58365  1.967636e-16        0.05421045
5         teb    17.90111  2.501886e-05        0.01453129
6        leaf   285.95672  9.100842e-58        0.19064332
```

6. Alternative Approach: Standardized Variables

Following the original analysis, let's also try the standardized approach:

```
# Create standardized variables
forest_standardized <- forest_df %>%
  mutate(
    npp_sqrt_scaled = scale(sqrt(npp))[,1],
    log_age_scaled = scale(log10(age))[,1],
    biomass_scaled = scale(biomass)[,1],
    temp_scaled = scale(temp)[,1],
    precip_scaled = scale(precip)[,1],
    teb_scaled = scale(teb)[,1]
  )

# Standardized model
model_std <- lm(npp_sqrt_scaled ~ log_age_scaled + biomass_scaled +
                temp_scaled * precip_scaled + teb_scaled,
                data = forest_standardized)

# Summary
summary(model_std)
```

```

Call:
lm(formula = npp_sqrt_scaled ~ log_age_scaled + biomass_scaled +
    temp_scaled * precip_scaled + teb_scaled, data = forest_standardized)

Residuals:
     Min       1Q   Median       3Q      Max
-3.14438 -0.38836 -0.03068  0.39178  3.01216

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)               -0.064952   0.020075  -3.235 0.001247 **
log_age_scaled            -0.543699   0.024500 -22.192  < 2e-16 ***
biomass_scaled             0.691415   0.025313  27.315  < 2e-16 ***
temp_scaled                0.215693   0.023304   9.256  < 2e-16 ***
precip_scaled             -0.008188   0.028714  -0.285 0.775563
teb_scaled                -0.071775   0.018879  -3.802 0.000151 ***
temp_scaled:precip_scaled  0.112727   0.017068   6.605 5.94e-11 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6113 on 1213 degrees of freedom
Multiple R-squared:  0.6281,    Adjusted R-squared:  0.6263
F-statistic: 341.5 on 6 and 1213 DF,  p-value: < 2.2e-16
```

7. Key Findings and Conclusions

1. MULTICOLLINEARITY:

   - Growing season length and temperature were highly correlated
   - Removed growing season to address multicollinearity

2. VARIABLE TRANSFORMATIONS:

   - Log transformation of age improved model fit
   - Cube root transformation of npp addressed assumption violations

3. FINAL MODEL RESULTS:

   - Significant predictors: age (negative), biomass (positive), temp (positive)
   - teb had negative effect, Leaf type differences were significant

4. BIOLOGICAL INTERPRETATION:

   - Younger stands had higher npp (for given biomass)
   - Higher biomass associated with higher npp
   - temp positively related to npp
   - Coniferous forests had lower npp than broadleaf forests

References and Additional Notes

This analysis is based on:

- **Michaletz, S.T., Cheng, D., Kerkhoff, A.J. & Enquist, B.J.** (2014). Convergence of terrestrial plant production across global climate gradients. *Nature*, 512, 39-43.

Key Learning Points:

1. **Multicollinearity Detection**: Use VIF values and correlation matrices
2. **Variable Transformations**: Log and power transformations can improve model fit
3. **Model Diagnostics**: Always check residual plots and assumption violations
4. **Model Comparison**: Use AIC and other criteria for model selection
5. **Interpretation**: Focus on biologically meaningful relationships

```r
# Session information for reproducibility
sessionInfo()
```

```
R version 4.5.1 (2025-06-13)
Platform: aarch64-apple-darwin20
```

```
Running under: macOS Sequoia 15.6

Matrix products: default
BLAS:   /Library/Frameworks/R.framework/Versions/4.5-arm64/Resources/lib/libRblas.0.dylib
LAPACK:   /Library/Frameworks/R.framework/Versions/4.5-arm64/Resources/lib/libRlapack.dylib;
LAPACK version 3.12.1

locale:
[1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8

time zone: America/Chicago
tzcode source: internal

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
 [1] sensemakr_0.1.6    see_0.11.0         performance_0.15.0 broom_1.0.9
 [5] GGally_2.3.0       corrplot_0.95      car_3.1-3          carData_3.0-5
 [9] lubridate_1.9.4    forcats_1.0.0      stringr_1.5.1      dplyr_1.1.4
[13] purrr_1.1.0        readr_2.1.5        tidyr_1.3.1        tibble_3.3.0
[17] ggplot2_3.5.2      tidyverse_2.0.0

loaded via a namespace (and not attached):
 [1] gtable_0.3.6       xfun_0.52          insight_1.3.1      lattice_0.22-7
 [5] tzdb_0.5.0         vctrs_0.6.5        tools_4.5.1        generics_0.1.4
 [9] parallel_4.5.1     pkgconfig_2.0.3    Matrix_1.7-3       RColorBrewer_1.1-3
[13] S7_0.2.0           lifecycle_1.0.4    compiler_4.5.1     farver_2.1.2
[17] codetools_0.2-20   htmltools_0.5.8.1  yaml_2.3.10        Formula_1.2-5
[21] pillar_1.11.0      crayon_1.5.3       abind_1.4-8        nlme_3.1-168
[25] ggstats_0.10.0     tidyselect_1.2.1   digest_0.6.37      stringi_1.8.7
[29] labeling_0.4.3     splines_4.5.1      fastmap_1.2.0      grid_4.5.1
[33] cli_3.6.5          magrittr_2.0.3     utf8_1.2.6         withr_3.0.2
[37] scales_1.4.0       backports_1.5.0    bit64_4.6.0-1      timechange_0.3.0
[41] rmarkdown_2.29     bit_4.6.0          hms_1.1.3          evaluate_1.0.4
[45] knitr_1.50         mgcv_1.9-3         rlang_1.1.6        glue_1.8.0
[49] rstudioapi_0.17.1  vroom_1.6.5        jsonlite_2.0.0     R6_2.6.1
```