Lecture 14 - Generalized Linear Models

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# Lecture 13: Review of ANOVAs

## Review

* ANOVA
* Factorial ANOVA
* Nested ANOVA
* ASSUMPTIONS OF ALL
  + Homogeneity of variance - Levenes or Bartlets Test
  + Normality of Residuals
  + Independence

NEED IMAGE FOR REVIEW

# Lecture 14: GLM Overview

## Overview

General Linear Models GLM

* Essentially the same as before while using defined distributions
  + Normal
  + Lognormal
  + Binomial
  + Poisson
  + Gamma
  + Negative binomial

Logistic Regression

* when the outcome is yes or no

# Overview of Generalized Linear Models (GLMs)

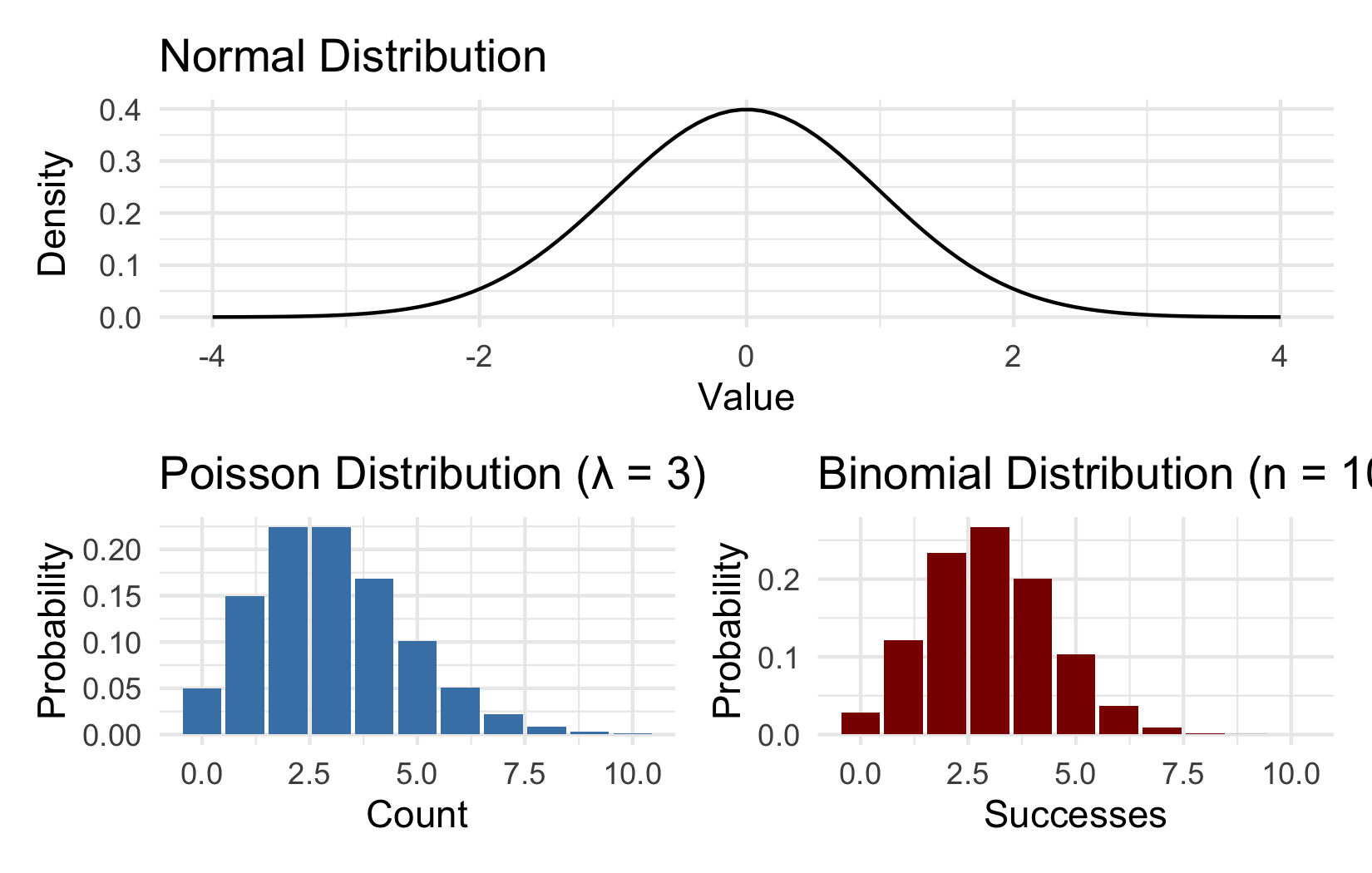
General linear models assume normal distribution of response variables and residuals. However, many types of biological data don’t meet this assumption. Generalized Linear Models (GLMs) allow for a wider range of probability distributions for the response variable.

GLMs allow all types of “exponential family” distributions:

* Normal
* Lognormal
* Binomial
* Poisson
* Gamma
* Negative binomial

GLMs can be used for binary (yes/no), discrete (count), and categorical/multinomial response variables, using maximum likelihood (ML) rather than ordinary least squares (OLS) for estimation.

**Note:** GLMs extend linear models to non-normal data distributions.



Examples of distributions in the exponential family

# The Three Elements of a GLM

GLMs consist of three components:

1. **Random component**: The response variable and its probability distribution (from exponential family: normal, binomial, Poisson)
2. **Systematic component**: The predictor variable(s) in the model, which can be continuous or categorical
3. **Link function**: Connects expected value of Y to predictor variables

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| Link Functions and Distributions |
| | Distribution | Common Link Function | Formula | | --- | --- | --- | | Normal | Identity |  | | Poisson | Log |  | | Binomial | Logit |  | |

# GLM with Gaussian (Normal) Distribution: Setup

The simplest form of GLM uses a normal (Gaussian) distribution with an identity link function. This is equivalent to standard linear regression.

Let’s compare a standard linear model and a Gaussian GLM using the mtcars dataset, modeling miles per gallon (mpg) by the number of cylinders (cyl).

# Convert cylinders to a factor  
mtcars <- mtcars %>%  
 mutate(cyl = factor(cyl))  
  
# Fit a standard linear model  
model\_lm <- lm(mpg ~ cyl, data = mtcars)  
  
# Fit a Gaussian GLM  
model\_gaussian <- glm(mpg ~ cyl,   
 data = mtcars,   
 family = gaussian(link = "identity"))  
  
# Compare the coefficients  
coef\_lm <- coefficients(model\_lm)  
coef\_glm <- coefficients(model\_gaussian)  
  
# Check if they're the same  
all.equal(coef\_lm, coef\_glm)

[1] TRUE

Let’s look at the summary of our Gaussian GLM:

summary(model\_gaussian)

Call:  
glm(formula = mpg ~ cyl, family = gaussian(link = "identity"),   
 data = mtcars)  
  
Coefficients:  
 Estimate Std. Error t value Pr(>|t|)   
(Intercept) 26.6636 0.9718 27.437 < 2e-16 \*\*\*  
cyl6 -6.9208 1.5583 -4.441 0.000119 \*\*\*  
cyl8 -11.5636 1.2986 -8.905 8.57e-10 \*\*\*  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  
(Dispersion parameter for gaussian family taken to be 10.38837)  
  
 Null deviance: 1126.05 on 31 degrees of freedom  
Residual deviance: 301.26 on 29 degrees of freedom  
AIC: 170.56  
  
Number of Fisher Scoring iterations: 2

# GLM with Gaussian Distribution: Analysis

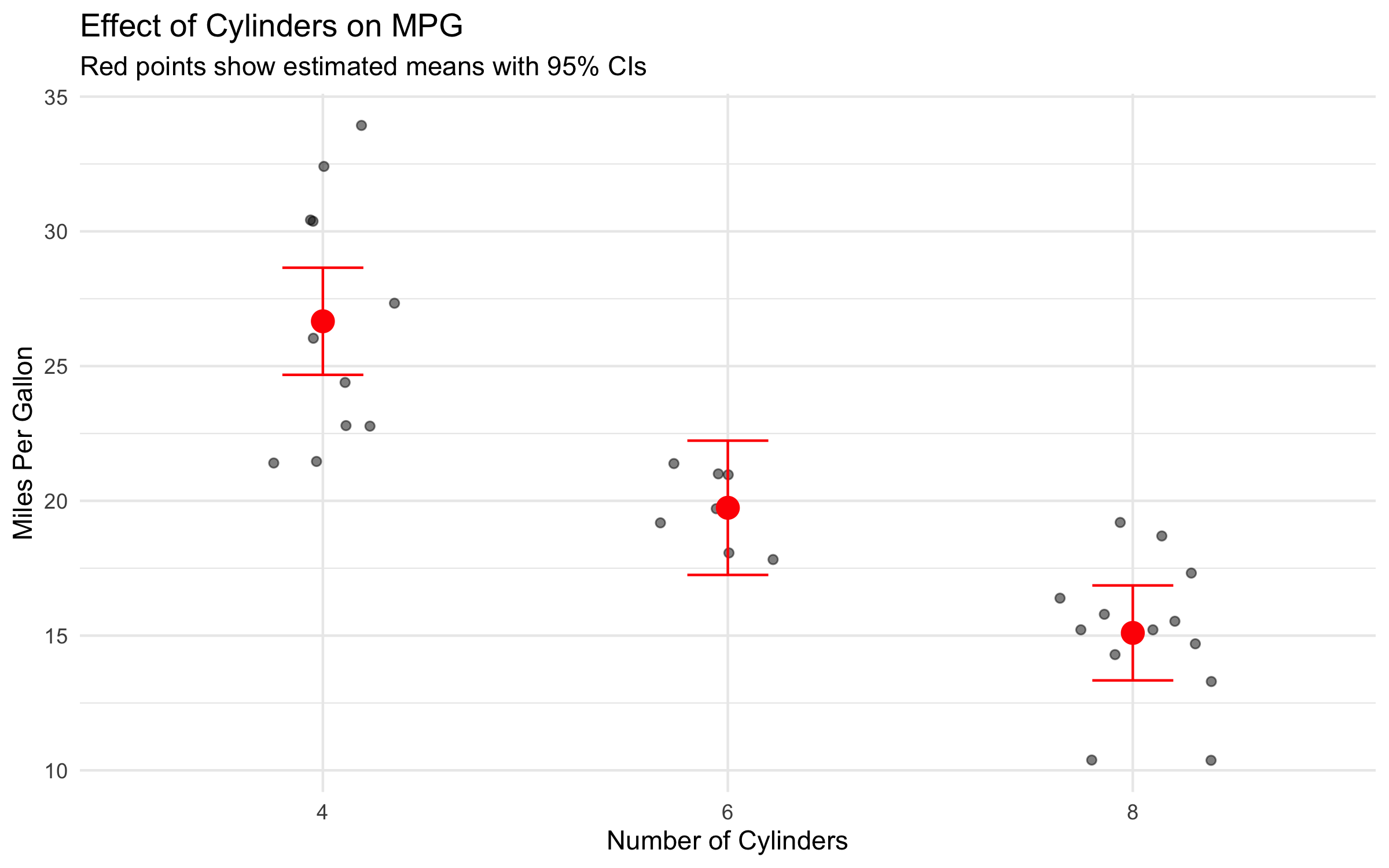
Now let’s perform an ANOVA on our GLM model using the car package:

Anova(model\_gaussian, type = "III", test = "F")

Analysis of Deviance Table (Type III tests)  
  
Response: mpg  
Error estimate based on Pearson residuals   
  
 Sum Sq Df F values Pr(>F)   
cyl 824.78 2 39.697 4.979e-09 \*\*\*  
Residuals 301.26 29   
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Visualizing the results:

# Get estimated means  
emm\_gaussian <- emmeans(model\_gaussian, ~ cyl)  
emm\_df <- as.data.frame(emm\_gaussian)  
  
# Create plot of data with estimated means  
ggplot() +  
 # Plot raw data  
 geom\_jitter(data = mtcars,   
 aes(x = cyl, y = mpg),   
 width = 0.2,   
 alpha = 0.5) +  
 # Add estimated means with confidence intervals  
 geom\_point(data = emm\_df,   
 aes(x = cyl, y = emmean),   
 size = 4, color = "red") +  
 geom\_errorbar(data = emm\_df,   
 aes(x = cyl,   
 ymin = lower.CL,   
 ymax = upper.CL),   
 width = 0.2,   
 color = "red") +  
 labs(title = "Effect of Cylinders on MPG",  
 subtitle = "Red points show estimated means with 95% CIs",  
 x = "Number of Cylinders",  
 y = "Miles Per Gallon") +  
 theme\_minimal()



# Equivalence of Linear Models and Gaussian GLMs

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| Equivalence of Linear Models and Gaussian GLMs |
| When we use a Gaussian distribution with an identity link, GLM gives identical results to standard linear regression. This can be seen in the coefficient values and overall model statistics.  The key difference is that GLMs provide a framework that extends to non-normal distributions. |

# GLM with Poisson Distribution: Setup

Poisson GLMs are appropriate for count data. The Poisson distribution assumes that the variance equals the mean.

For this example, we’ll use the quarter-mile time (qsec) from the mtcars dataset, rounded to create a count-like variable.

# Prepare data for Poisson model  
mtcars\_count <- mtcars %>%  
 mutate(  
 cyl = factor(cyl),  
 qsec\_round = round(qsec) # Create a count-like variable  
 )  
  
# Look at the first few rows  
head(mtcars\_count[, c("cyl", "qsec", "qsec\_round")])

cyl qsec qsec\_round  
Mazda RX4 6 16.46 16  
Mazda RX4 Wag 6 17.02 17  
Datsun 710 4 18.61 19  
Hornet 4 Drive 6 19.44 19  
Hornet Sportabout 8 17.02 17  
Valiant 6 20.22 20

Now let’s fit a Poisson GLM to model the relationship between the rounded quarter-mile time and the number of cylinders:

# Fit a Poisson GLM  
model\_poisson <- glm(qsec\_round ~ cyl,   
 family = poisson(link = "log"),   
 data = mtcars\_count)  
  
# Look at the model summary  
summary(model\_poisson)

Call:  
glm(formula = qsec\_round ~ cyl, family = poisson(link = "log"),   
 data = mtcars\_count)  
  
Coefficients:  
 Estimate Std. Error z value Pr(>|z|)   
(Intercept) 2.95869 0.06868 43.079 <2e-16 \*\*\*  
cyl6 -0.07629 0.11277 -0.676 0.499   
cyl8 -0.14243 0.09482 -1.502 0.133   
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  
(Dispersion parameter for poisson family taken to be 1)  
  
 Null deviance: 5.6979 on 31 degrees of freedom  
Residual deviance: 3.4487 on 29 degrees of freedom  
AIC: 160.62  
  
Number of Fisher Scoring iterations: 3

Let’s check for overdispersion, which is common in count data:

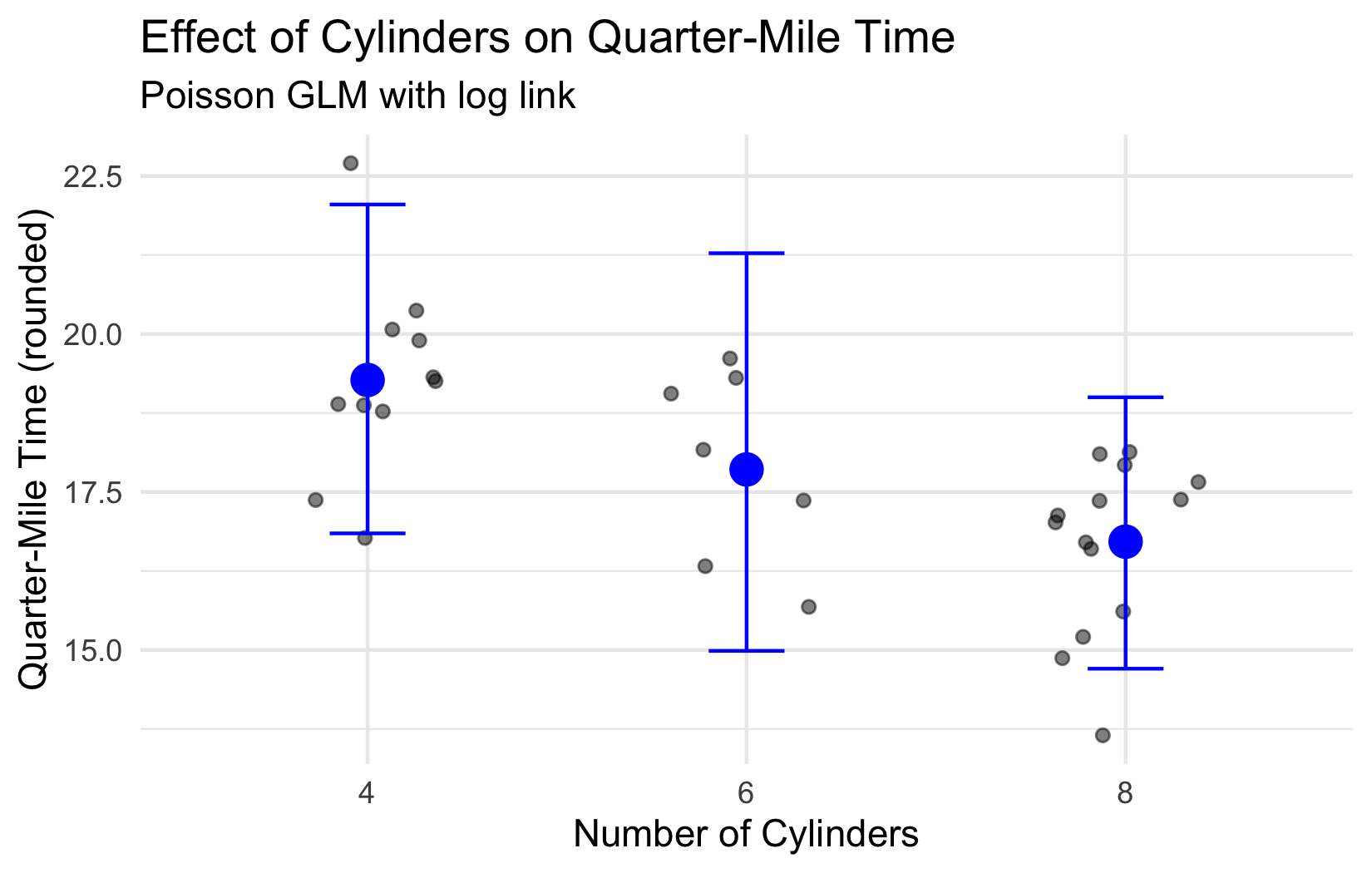
# Calculate dispersion parameter  
dispersion\_poisson <- sum(residuals(model\_poisson,   
 type = "pearson")^2) /   
 model\_poisson$df.residual  
  
# Print dispersion parameter  
cat("Dispersion parameter:", round(dispersion\_poisson, 2), "\n")

Dispersion parameter: 0.12

# Should be close to 1 for a well-fitting Poisson model  
# If > 1.5, may indicate overdispersion

# Poisson GLM: Visualization and Interpretation

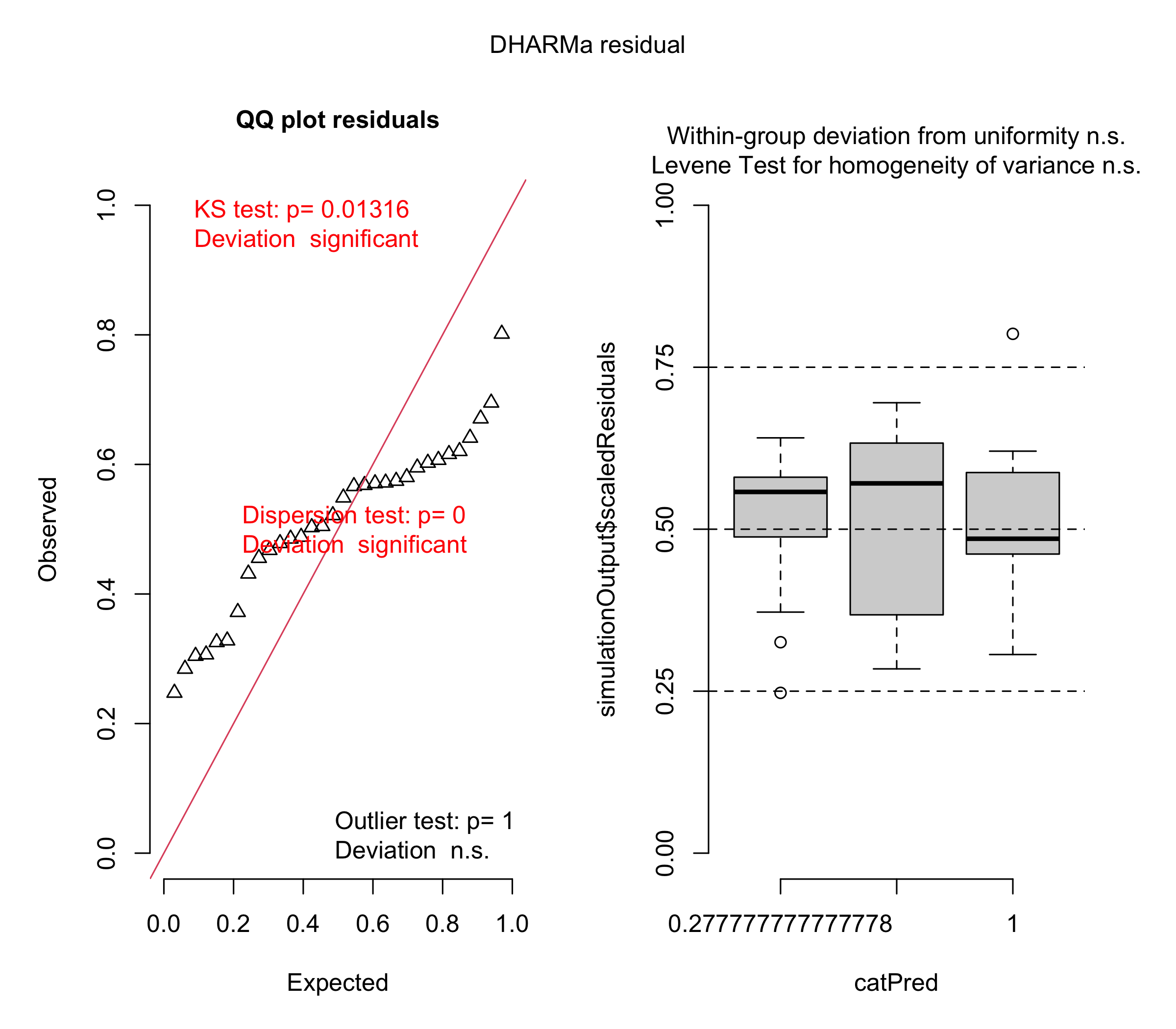
# Get estimated means on the response scale  
emm\_poisson <- emmeans(model\_poisson, ~ cyl, type = "response")  
emm\_poisson\_df <- as.data.frame(emm\_poisson)  
  
# Create visualization  
ggplot() +  
 # Plot raw data  
 geom\_jitter(data = mtcars\_count,   
 aes(x = cyl, y = qsec\_round),   
 width = 0.2,   
 alpha = 0.5) +  
 # Add estimated means with confidence intervals  
 geom\_point(data = emm\_poisson\_df,   
 aes(x = cyl, y = rate),   
 size = 4, color = "blue") +  
 geom\_errorbar(data = emm\_poisson\_df,   
 aes(x = cyl,   
 ymin = asymp.LCL,   
 ymax = asymp.UCL),   
 width = 0.2,   
 color = "blue") +  
 labs(title = "Effect of Cylinders on Quarter-Mile Time",  
 subtitle = "Poisson GLM with log link",  
 x = "Number of Cylinders",  
 y = "Quarter-Mile Time (rounded)") +  
 theme\_minimal()



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| Interpreting Poisson GLM Coefficients |
| In a Poisson GLM with a log link function:   1. The coefficients represent changes in the **log** of the expected count 2. When exponentiated (exp(coef)), they represent multiplicative effects 3. For example, exp(coef) = 0.90 means the expected count is 90% of the reference level |

# Checking Model Assumptions with DHARMa

# Simulate residuals using DHARMa  
set.seed(123) # For reproducibility  
simulation\_poisson <- simulateResiduals(fittedModel = model\_poisson, n = 1000)  
  
# Plot diagnostic plots  
plot(simulation\_poisson)



# Dealing with Overdispersion in Count Data

When count data shows more variability than expected under a Poisson distribution (variance > mean), we may need to use a negative binomial model instead.

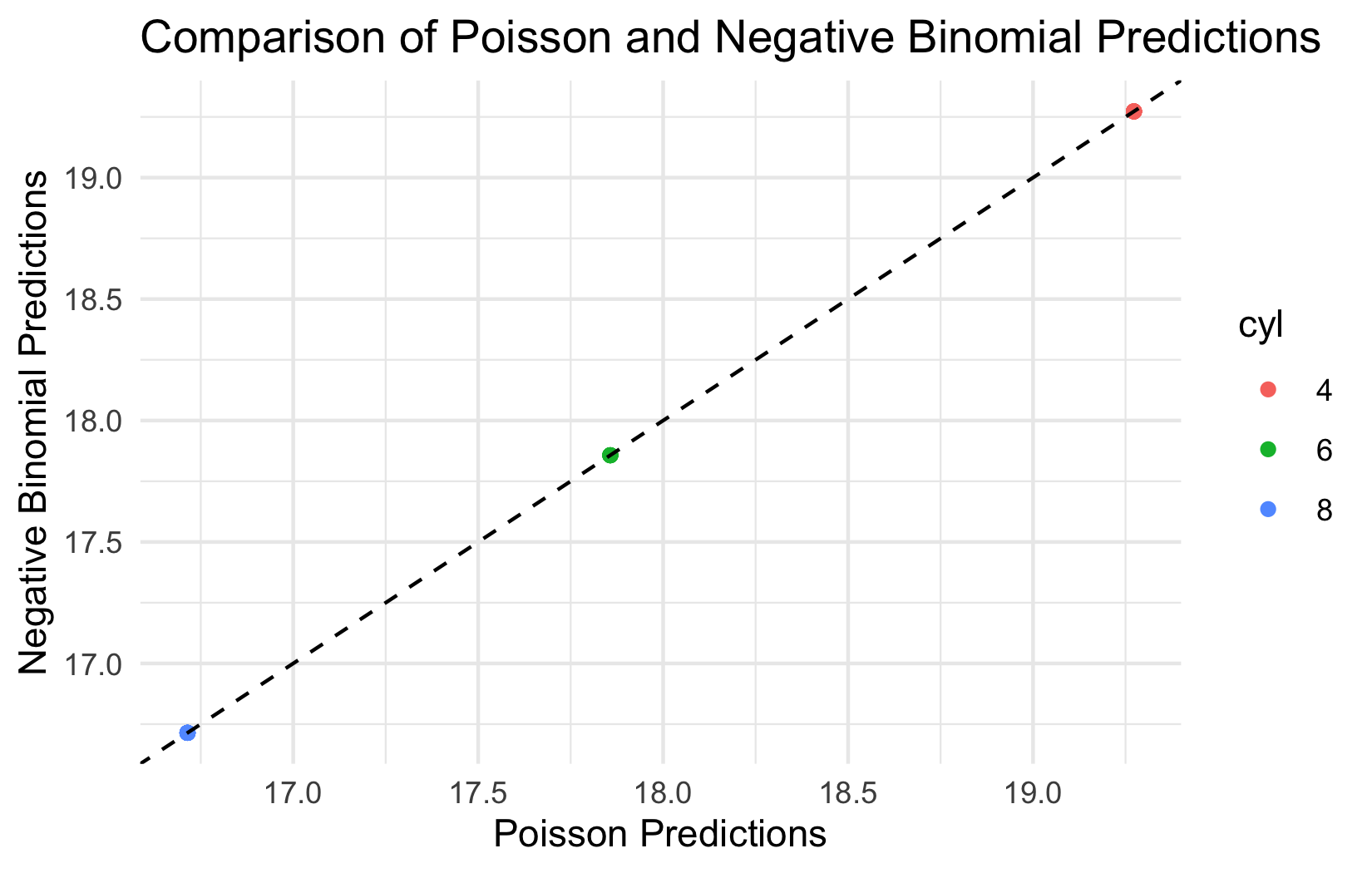
# If we detected overdispersion, we could fit a negative binomial model  
# This is just for demonstration - our data may not actually need this  
  
# Fit negative binomial model  
model\_nb <- glm.nb(qsec\_round ~ cyl, data = mtcars\_count)  
  
# Compare summaries  
summary(model\_nb)

Call:  
glm.nb(formula = qsec\_round ~ cyl, data = mtcars\_count, init.theta = 2935650.009,   
 link = log)  
  
Coefficients:  
 Estimate Std. Error z value Pr(>|z|)   
(Intercept) 2.95869 0.06868 43.079 <2e-16 \*\*\*  
cyl6 -0.07629 0.11277 -0.676 0.499   
cyl8 -0.14243 0.09482 -1.502 0.133   
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  
(Dispersion parameter for Negative Binomial(2935650) family taken to be 1)  
  
 Null deviance: 5.6979 on 31 degrees of freedom  
Residual deviance: 3.4486 on 29 degrees of freedom  
AIC: 162.62  
  
Number of Fisher Scoring iterations: 1  
  
 Theta: 2935650   
 Std. Err.: 121368753   
Warning while fitting theta: iteration limit reached   
  
 2 x log-likelihood: -154.616

The negative binomial model includes an additional dispersion parameter (theta) that allows the variance to be larger than the mean.

Let’s compare the predictions from both models:

# Create predictions from both models  
mtcars\_count$pred\_poisson <- predict(model\_poisson,   
 type = "response")  
mtcars\_count$pred\_nb <- predict(model\_nb,   
 type = "response")  
  
# Compare predictions  
ggplot(mtcars\_count) +  
 geom\_point(aes(x = pred\_poisson, y = pred\_nb, color = cyl)) +  
 geom\_abline(intercept = 0, slope = 1, linetype = "dashed") +  
 labs(title = "Comparison of Poisson and Negative Binomial Predictions",  
 x = "Poisson Predictions",  
 y = "Negative Binomial Predictions") +  
 theme\_minimal()



# Logistic Regression - Introduction

Logistic regression is a GLM used when the response variable is binary (e.g., dead/alive, present/absent). It models the probability of the response being “1” (success) given predictor values.

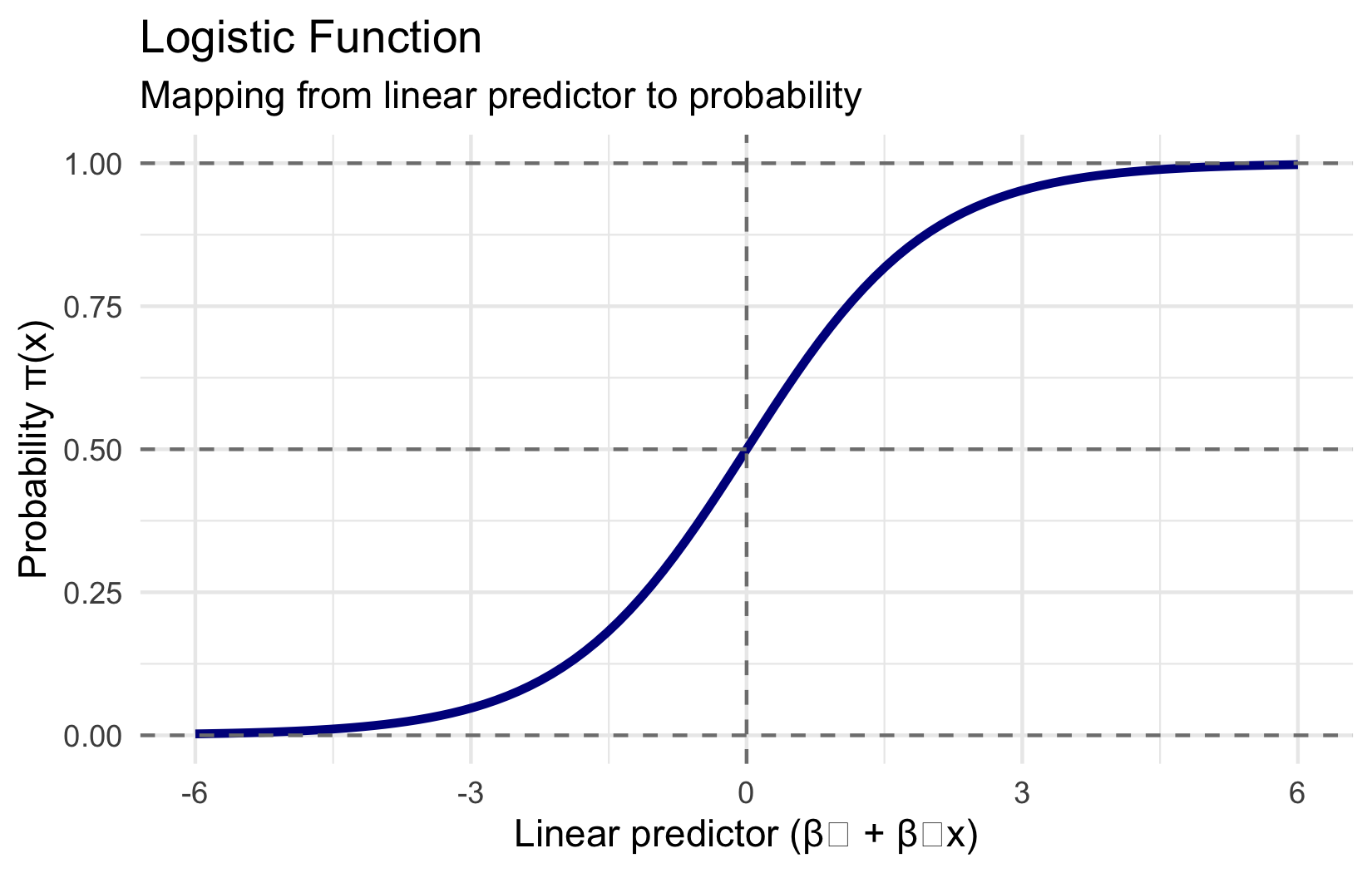
Let’s examine the simple logistic regression model:

Where: - is the probability that Y = 1 given X = x - is the intercept - is the slope (rate of change in for a unit change in X)

To linearize this relationship, we use the logit link function:

This transforms the probability (which is bounded between 0 and 1) to a linear function that can range from -∞ to +∞.

# Create data for sigmoid curve  
sigmoid\_data <- data.frame(  
 x = seq(-6, 6, length.out = 100)  
)  
sigmoid\_data$p <- 1 / (1 + exp(-sigmoid\_data$x))  
  
# Plot the sigmoid curve  
ggplot(sigmoid\_data, aes(x, p)) +  
 geom\_line(linewidth = 1.2, color = "darkblue") +  
 geom\_hline(yintercept = c(0, 0.5, 1),   
 linetype = "dashed",   
 color = "gray50") +  
 geom\_vline(xintercept = 0,   
 linetype = "dashed",   
 color = "gray50") +  
 labs(title = "Logistic Function",  
 subtitle = "Mapping from linear predictor to probability",  
 x = "Linear predictor (β₀ + β₁x)",  
 y = "Probability π(x)") +  
 scale\_y\_continuous(breaks = seq(0, 1, 0.25)) +  
 theme\_minimal()



# Example: Lizard Presence on Islands

Based on the example from Polis et al. (1998), we’ll model the presence/absence of lizards (*Uta*) on islands in the Gulf of California based on perimeter/area ratio.

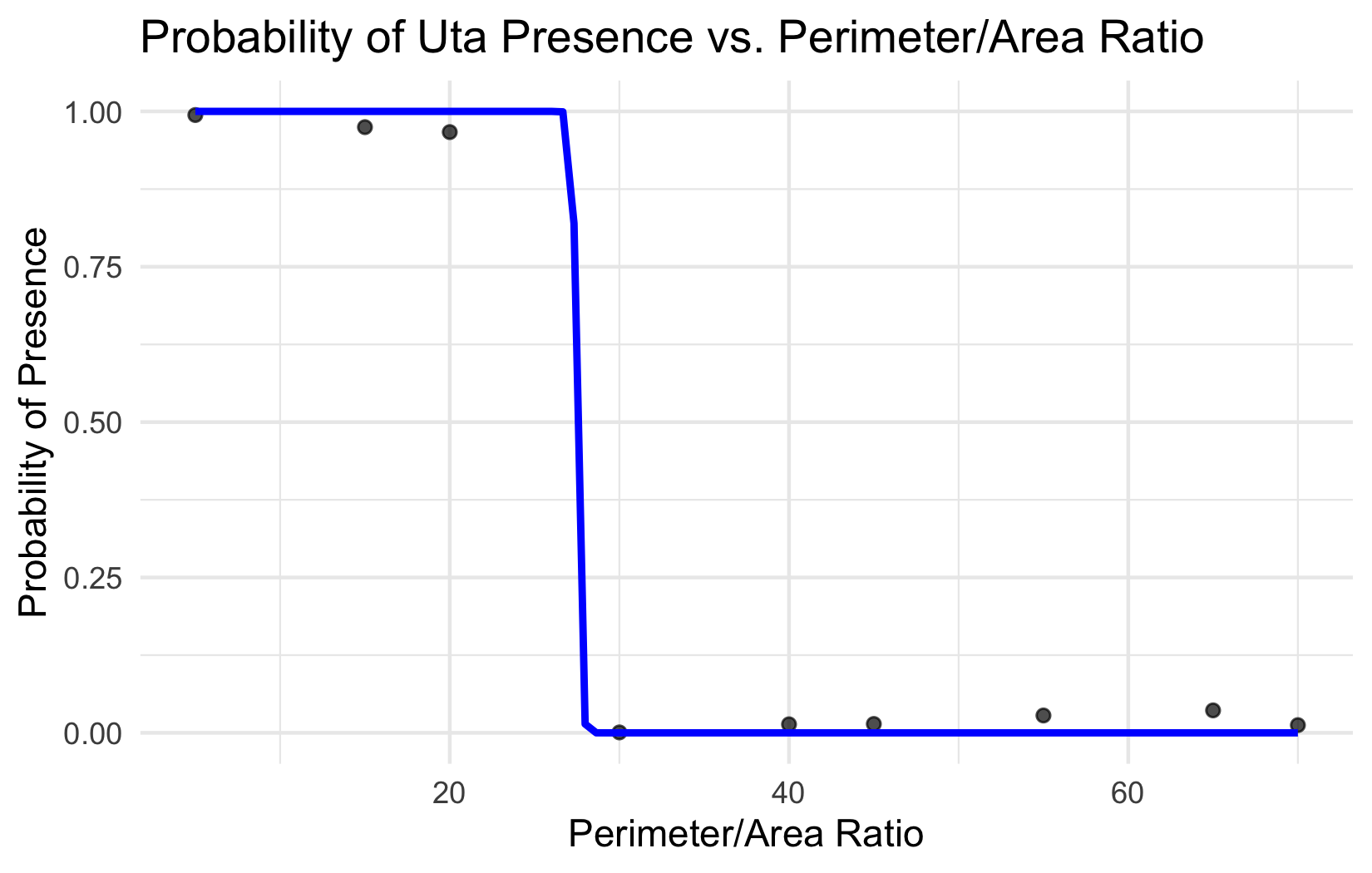
# Create a simulated dataset based on the described study  
set.seed(123)  
island\_data <- data.frame(  
 island\_id = 1:19,  
 pa\_ratio = c(5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 10, 15, 20, 25, 30),  
 uta\_present = c(1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0)  
) %>%  
 mutate(uta\_present = factor(uta\_present, levels = c(0, 1), labels = c("Absent", "Present")))  
  
# Fit the logistic regression model  
lizard\_model <- glm(uta\_present ~ pa\_ratio,   
 data = island\_data,   
 family = binomial(link = "logit"))  
  
# Model summary  
summary(lizard\_model)

Call:  
glm(formula = uta\_present ~ pa\_ratio, family = binomial(link = "logit"),   
 data = island\_data)  
  
Coefficients:  
 Estimate Std. Error z value Pr(>|z|)  
(Intercept) 241.039 191755.596 0.001 0.999  
pa\_ratio -8.766 6965.289 -0.001 0.999  
  
(Dispersion parameter for binomial family taken to be 1)  
  
 Null deviance: 2.6287e+01 on 18 degrees of freedom  
Residual deviance: 2.4292e-09 on 17 degrees of freedom  
AIC: 4  
  
Number of Fisher Scoring iterations: 25

# Lizard Example: Visualization and Testing

Let’s visualize the data and the fitted model:

# Create a dataframe for predictions  
pred\_data <- data.frame(  
 pa\_ratio = seq(min(island\_data$pa\_ratio),   
 max(island\_data$pa\_ratio),   
 length.out = 100)  
)  
  
# Get predicted probabilities  
pred\_data$prob <- predict(lizard\_model,   
 newdata = pred\_data,   
 type = "response")  
  
# Plot  
ggplot() +  
 # Add jittered points for observed data  
 geom\_jitter(data = island\_data,   
 aes(x = pa\_ratio, y = as.numeric(uta\_present) - 1),  
 height = 0.05, width = 0, alpha = 0.7) +  
 # Add predicted probability curve  
 geom\_line(data = pred\_data,   
 aes(x = pa\_ratio, y = prob),   
 color = "blue", size = 1) +  
 # Add confidence intervals (optional)  
 labs(title = "Probability of Uta Presence vs. Perimeter/Area Ratio",  
 x = "Perimeter/Area Ratio",  
 y = "Probability of Presence") +  
 scale\_y\_continuous(limits = c(0, 1)) +  
 theme\_minimal()



We want to test the null hypothesis that β₁ = 0, meaning there’s no relationship between P/A ratio and lizard presence.

There are two common ways to test this hypothesis:

1. **Wald test**: Tests if the parameter estimate divided by its standard error differs significantly from zero
2. **Likelihood ratio test**: Compares the fit of the full model to a reduced model without the predictor variable

# Reduced model (intercept only)  
reduced\_model <- glm(uta\_present ~ 1,   
 data = island\_data,   
 family = binomial(link = "logit"))  
  
# Likelihood ratio test  
anova(reduced\_model, lizard\_model, test = "Chisq")

Analysis of Deviance Table  
  
Model 1: uta\_present ~ 1  
Model 2: uta\_present ~ pa\_ratio  
 Resid. Df Resid. Dev Df Deviance Pr(>Chi)   
1 18 26.287   
2 17 0.000 1 26.287 2.943e-07 \*\*\*  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

# Interpreting the Odds Ratio

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| Working with Odds Ratios |
| The odds ratio represents how the odds of the event (e.g., lizard presence) change with a unit increase in the predictor.   * **Odds ratio = exp(β₁)** * If odds ratio > 1: Increasing the predictor increases the odds of event * If odds ratio < 1: Increasing the predictor decreases the odds of event * If odds ratio = 1: No effect of predictor on odds of event |

# Calculate odds ratio and confidence interval  
coef\_lizard <- coef(lizard\_model)[2] # Extract slope coefficient  
odds\_ratio <- exp(coef\_lizard)  
ci <- exp(confint(lizard\_model, "pa\_ratio"))  
  
# Display results  
cat("Odds Ratio:", round(odds\_ratio, 3), "\n")

Odds Ratio: 0

cat("95% CI:", round(ci[1], 3), "to", round(ci[2], 3), "\n")

95% CI: 0 to Inf

# Assessing Model Fit

There are several ways to assess the goodness-of-fit for logistic regression models:

# Calculate Hosmer-Lemeshow statistic  
# This would normally require an additional package like 'ResourceSelection'  
# Instead, we'll use a simpler approximation and other diagnostics  
  
# Calculate Pearson residuals  
pearson\_resid <- residuals(lizard\_model, type = "pearson")  
pearson\_chi2 <- sum(pearson\_resid^2)  
df\_resid <- lizard\_model$df.residual  
  
# Calculate deviance  
deviance\_g2 <- lizard\_model$deviance  
null\_deviance <- lizard\_model$null.deviance  
  
# Calculate McFadden's pseudo-R²  
r2\_mcfadden <- 1 - (deviance\_g2 / null\_deviance)  
  
# Display results  
cat("Pearson χ²:", round(pearson\_chi2, 3), "on", df\_resid, "df, p =",   
 round(1 - pchisq(pearson\_chi2, df\_resid), 3), "\n")

Pearson χ²: 0 on 17 df, p = 1

cat("Deviance G²:", round(deviance\_g2, 3), "on", df\_resid, "df, p =",   
 round(1 - pchisq(deviance\_g2, df\_resid), 3), "\n")

Deviance G²: 0 on 17 df, p = 1

cat("McFadden's R²:", round(r2\_mcfadden, 3), "\n")

McFadden's R²: 1

# Multiple Logistic Regression: Setup

Logistic regression can be extended to include multiple predictors. The model becomes:

Where g(x) is the logit link function, and x₁, x₂, …, xₚ are the predictor variables.

Let’s create a simulated dataset based on the Bolger et al. (1997) study of the presence/absence of native rodents in canyon fragments.

# Simulate data for the rodent example  
set.seed(123)  
n <- 25 # 25 canyon fragments  
  
# Create predictor variables  
fragment\_data <- data.frame(  
 fragment\_id = paste0("F", 1:n),  
 distance = runif(n, 0, 3000), # Distance to source canyon (m)  
 age = runif(n, 5, 80), # Years since isolation  
 shrub\_cover = runif(n, 10, 100) # Percentage shrub cover  
)  
  
# Generate response variable (rodent presence)  
# Higher probability with higher shrub cover, slight effect of age  
linear\_pred <- -5 + 0.0001\*fragment\_data$distance +   
 0.02\*fragment\_data$age +   
 0.09\*fragment\_data$shrub\_cover  
prob <- 1 / (1 + exp(-linear\_pred))  
fragment\_data$rodent\_present <- rbinom(n, 1, prob)  
fragment\_data$rodent\_present <- factor(fragment\_data$rodent\_present,   
 levels = c(0, 1),   
 labels = c("Absent", "Present"))  
  
# Fit multiple logistic regression model  
rodent\_model <- glm(rodent\_present ~ distance + age + shrub\_cover,   
 data = fragment\_data,   
 family = binomial(link = "logit"))  
  
# Model summary  
summary(rodent\_model)

Call:  
glm(formula = rodent\_present ~ distance + age + shrub\_cover,   
 family = binomial(link = "logit"), data = fragment\_data)  
  
Coefficients:  
 Estimate Std. Error z value Pr(>|z|)   
(Intercept) -12.278261 7.911491 -1.552 0.1207   
distance 0.002062 0.001716 1.202 0.2294   
age 0.068744 0.059665 1.152 0.2493   
shrub\_cover 0.193001 0.116035 1.663 0.0963 .  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  
(Dispersion parameter for binomial family taken to be 1)  
  
 Null deviance: 27.5540 on 24 degrees of freedom  
Residual deviance: 9.2737 on 21 degrees of freedom  
AIC: 17.274  
  
Number of Fisher Scoring iterations: 8

To test the significance of individual predictors, we can use likelihood ratio tests comparing nested models:

# Test distance  
model\_no\_distance <- glm(rodent\_present ~ age + shrub\_cover,   
 data = fragment\_data,   
 family = binomial(link = "logit"))  
anova(model\_no\_distance, rodent\_model, test = "Chisq")

Analysis of Deviance Table  
  
Model 1: rodent\_present ~ age + shrub\_cover  
Model 2: rodent\_present ~ distance + age + shrub\_cover  
 Resid. Df Resid. Dev Df Deviance Pr(>Chi)  
1 22 11.3831   
2 21 9.2737 1 2.1094 0.1464

# Test age  
model\_no\_age <- glm(rodent\_present ~ distance + shrub\_cover,   
 data = fragment\_data,   
 family = binomial(link = "logit"))  
anova(model\_no\_age, rodent\_model, test = "Chisq")

Analysis of Deviance Table  
  
Model 1: rodent\_present ~ distance + shrub\_cover  
Model 2: rodent\_present ~ distance + age + shrub\_cover  
 Resid. Df Resid. Dev Df Deviance Pr(>Chi)  
1 22 11.0533   
2 21 9.2737 1 1.7796 0.1822

# Test shrub cover  
model\_no\_shrub <- glm(rodent\_present ~ distance + age,   
 data = fragment\_data,   
 family = binomial(link = "logit"))  
anova(model\_no\_shrub, rodent\_model, test = "Chisq")

Analysis of Deviance Table  
  
Model 1: rodent\_present ~ distance + age  
Model 2: rodent\_present ~ distance + age + shrub\_cover  
 Resid. Df Resid. Dev Df Deviance Pr(>Chi)   
1 22 26.7315   
2 21 9.2737 1 17.458 2.938e-05 \*\*\*  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

# Multiple Logistic Regression: Odds Ratios

Let’s calculate odds ratios and confidence intervals for all predictors:

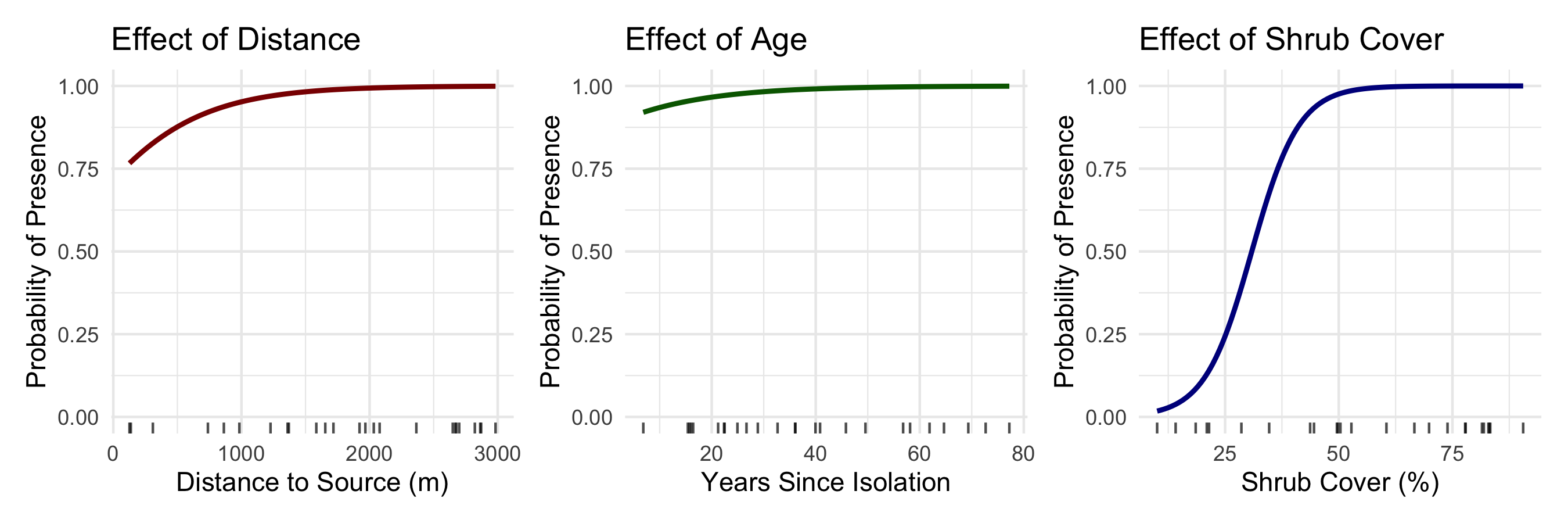
# Calculate odds ratios and CIs  
coefs <- coef(rodent\_model)[-1] # Exclude intercept  
odds\_ratios <- exp(coefs)  
ci <- exp(confint(rodent\_model)[-1, ]) # Exclude intercept  
  
# Create a data frame for display  
or\_df <- data.frame(  
 Predictor = names(coefs),  
 OddsRatio = odds\_ratios,  
 LowerCI = ci[, 1],  
 UpperCI = ci[, 2]  
)  
  
# Display formatted table  
or\_df %>%  
 mutate(across(where(is.numeric), round, 4)) %>%  
 mutate(CI = paste0("(", LowerCI, ", ", UpperCI, ")")) %>%  
 dplyr::select(Predictor, OddsRatio, CI) %>%  
 flextable()

| **Predictor** | **OddsRatio** | **CI** |
| --- | --- | --- |
| distance | 1.0021 | (0.9994, 1.0069) |
| age | 1.0712 | (0.9721, 1.2577) |
| shrub\_cover | 1.2129 | (1.0645, 1.7909) |

# Visualizing Multiple Logistic Regression

For multiple predictors, we can visualize the effect of each predictor while holding others constant at their mean or median values.

# Create a function to generate prediction data for one variable  
predict\_for\_var <- function(var\_name, model, data) {  
 # Create grid of values for the variable of interest  
 pred\_df <- data.frame(  
 x = seq(min(data[[var\_name]]), max(data[[var\_name]]), length.out = 100)  
 )  
 names(pred\_df) <- var\_name  
   
 # Add mean values for other predictors  
 for (other\_var in c("distance", "age", "shrub\_cover")) {  
 if (other\_var != var\_name) {  
 pred\_df[[other\_var]] <- mean(data[[other\_var]])  
 }  
 }  
   
 # Add predictions  
 pred\_df$prob <- predict(model, newdata = pred\_df, type = "response")  
   
 return(pred\_df)  
}  
  
# Generate prediction data for each variable  
pred\_distance <- predict\_for\_var("distance", rodent\_model, fragment\_data)  
pred\_age <- predict\_for\_var("age", rodent\_model, fragment\_data)  
pred\_shrub <- predict\_for\_var("shrub\_cover", rodent\_model, fragment\_data)  
  
# Create plots  
p1 <- ggplot() +  
 geom\_rug(data = fragment\_data,   
 aes(x = distance, y = as.numeric(rodent\_present) - 1),  
 sides = "b", alpha = 0.7) +  
 geom\_line(data = pred\_distance, aes(x = distance, y = prob),   
 color = "darkred", size = 1) +  
 labs(title = "Effect of Distance",  
 x = "Distance to Source (m)",  
 y = "Probability of Presence") +  
 theme\_minimal()  
  
p2 <- ggplot() +  
 geom\_rug(data = fragment\_data,   
 aes(x = age, y = as.numeric(rodent\_present) - 1),  
 sides = "b", alpha = 0.7) +  
 geom\_line(data = pred\_age, aes(x = age, y = prob),   
 color = "darkgreen", size = 1) +  
 labs(title = "Effect of Age",  
 x = "Years Since Isolation",  
 y = "Probability of Presence") +  
 theme\_minimal()  
  
p3 <- ggplot() +  
 geom\_rug(data = fragment\_data,   
 aes(x = shrub\_cover, y = as.numeric(rodent\_present) - 1),  
 sides = "b", alpha = 0.7) +  
 geom\_line(data = pred\_shrub, aes(x = shrub\_cover, y = prob),   
 color = "darkblue", size = 1) +  
 labs(title = "Effect of Shrub Cover",  
 x = "Shrub Cover (%)",  
 y = "Probability of Presence") +  
 theme\_minimal()  
  
# Combine plots  
p1 + p2 + p3



This visualization shows the effect of each predictor on the probability of rodent presence, while holding the other predictors constant at their mean values.

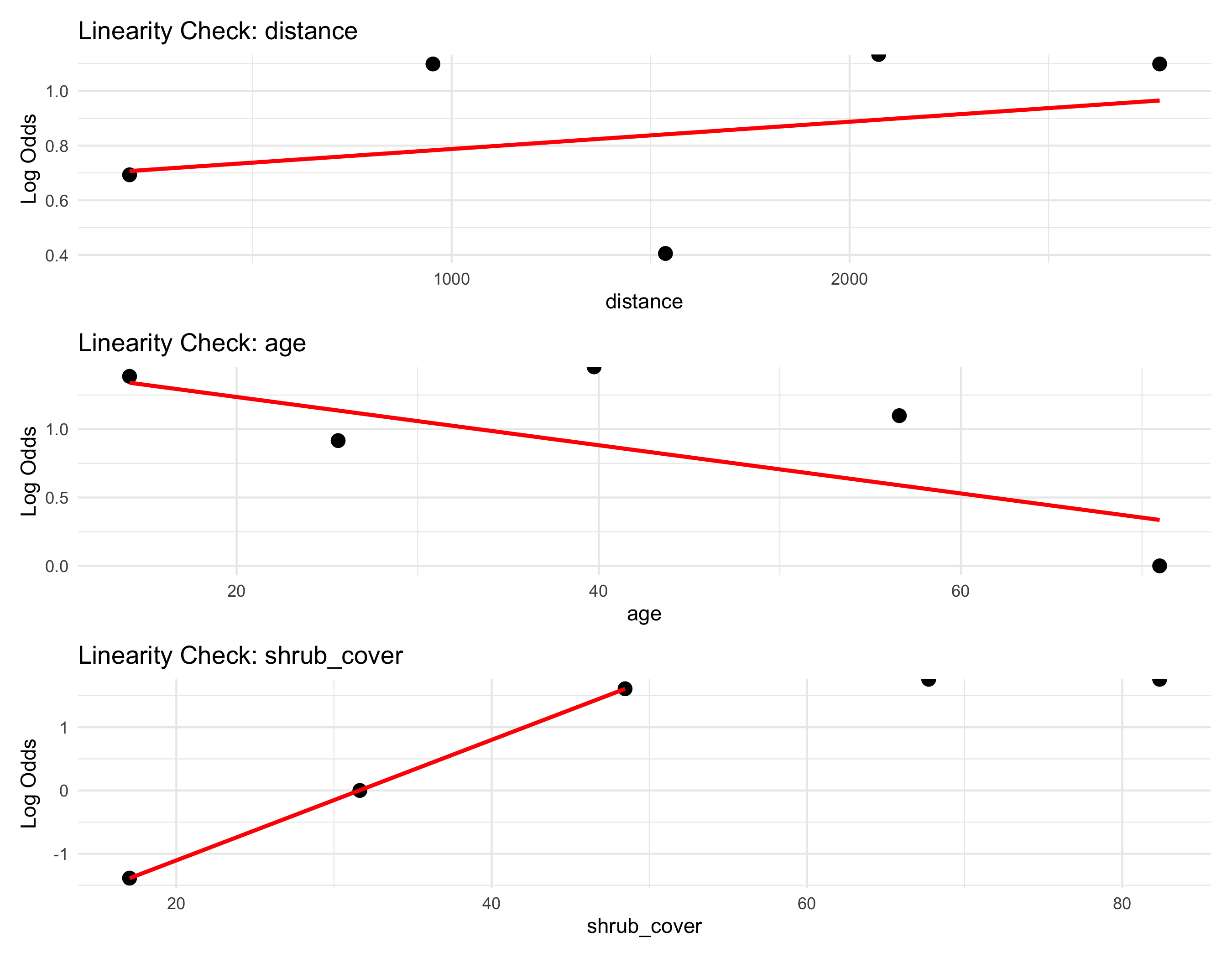
# Assumptions and Diagnostics of Logistic Regression

Logistic regression has several key assumptions:

1. Independence of observations
2. Linear relationship between predictors and log odds
3. No extreme outliers
4. No multicollinearity (when multiple predictors are used)

Let’s check the diagnostics for our multiple logistic regression model:

# 1. Check for linearity between predictors and log odds  
# Use bins of X variables and plot log odds  
check\_linearity <- function(model, data, var) {  
 # Create bins of predictor  
 n\_bins <- 5  
 data$bin <- cut(data[[var]], breaks = n\_bins)  
   
 # Calculate log odds for each bin  
 bin\_summary <- data %>%  
 group\_by(bin) %>%  
 summarize(  
 n = n(),  
 mean\_var = mean(!!sym(var)),  
 successes = sum(rodent\_present == "Present"),  
 failures = sum(rodent\_present == "Absent")  
 ) %>%  
 mutate(  
 p = successes / n,  
 logodds = log(p / (1 - p))  
 )  
   
 # Create plot  
 ggplot(bin\_summary, aes(x = mean\_var, y = logodds)) +  
 geom\_point(size = 3) +  
 geom\_smooth(method = "lm", se = FALSE, color = "red") +  
 labs(title = paste("Linearity Check:", var),  
 x = var,  
 y = "Log Odds") +  
 theme\_minimal()  
}  
  
# Create diagnostic plots for each variable  
p1 <- check\_linearity(rodent\_model, fragment\_data, "distance")  
p2 <- check\_linearity(rodent\_model, fragment\_data, "age")  
p3 <- check\_linearity(rodent\_model, fragment\_data, "shrub\_cover")  
  
# Arrange the plots  
p1 / p2 / p3



# Model Comparison and Selection

When working with multiple predictors, we often want to find the most parsimonious model. We can use:

1. Likelihood ratio tests for nested models
2. Information criteria (AIC, BIC) for non-nested models
3. Classification metrics like accuracy, sensitivity, and specificity

Let’s compare models and calculate AIC values:

# Calculate AIC for our models  
models <- list(  
 "Full" = rodent\_model,  
 "No Distance" = model\_no\_distance,  
 "No Age" = model\_no\_age,  
 "No Shrub" = model\_no\_shrub,  
 "Intercept Only" = glm(rodent\_present ~ 1,   
 data = fragment\_data,   
 family = binomial)  
)  
  
# Calculate AIC and BIC  
model\_comparison <- data.frame(  
 Model = names(models),  
 Parameters = sapply(models, function(m) length(coef(m))),  
 AIC = sapply(models, AIC),  
 BIC = sapply(models, BIC),  
 Deviance = sapply(models, function(m) m$deviance)  
)  
  
# Show model comparison table  
model\_comparison %>%  
 arrange(AIC) %>%  
 mutate(across(where(is.numeric), round, 2)) %>%  
 flextable()

| **Model** | **Parameters** | **AIC** | **BIC** | **Deviance** |
| --- | --- | --- | --- | --- |
| No Age | 3 | 17.05 | 20.71 | 11.05 |
| Full | 4 | 17.27 | 22.15 | 9.27 |
| No Distance | 3 | 17.38 | 21.04 | 11.38 |
| Intercept Only | 1 | 29.55 | 30.77 | 27.55 |
| No Shrub | 3 | 32.73 | 36.39 | 26.73 |

We can also evaluate the predictive performance of our model:

# Get predictions  
predicted\_probs <- predict(rodent\_model, type = "response")  
predicted\_class <- ifelse(predicted\_probs > 0.5, "Present", "Absent")  
  
# Create confusion matrix  
true\_class <- fragment\_data$rodent\_present  
conf\_matrix <- table(Predicted = predicted\_class, Actual = true\_class)  
  
# Calculate metrics  
accuracy <- sum(diag(conf\_matrix)) / sum(conf\_matrix)  
sensitivity <- conf\_matrix["Present", "Present"] / sum(conf\_matrix[, "Present"])  
specificity <- conf\_matrix["Absent", "Absent"] / sum(conf\_matrix[, "Absent"])  
  
# Display results  
conf\_matrix

Actual  
Predicted Absent Present  
 Absent 5 2  
 Present 1 17

cat("\nAccuracy:", round(accuracy, 3), "\n")

Accuracy: 0.88

cat("Sensitivity:", round(sensitivity, 3), "\n")

Sensitivity: 0.895

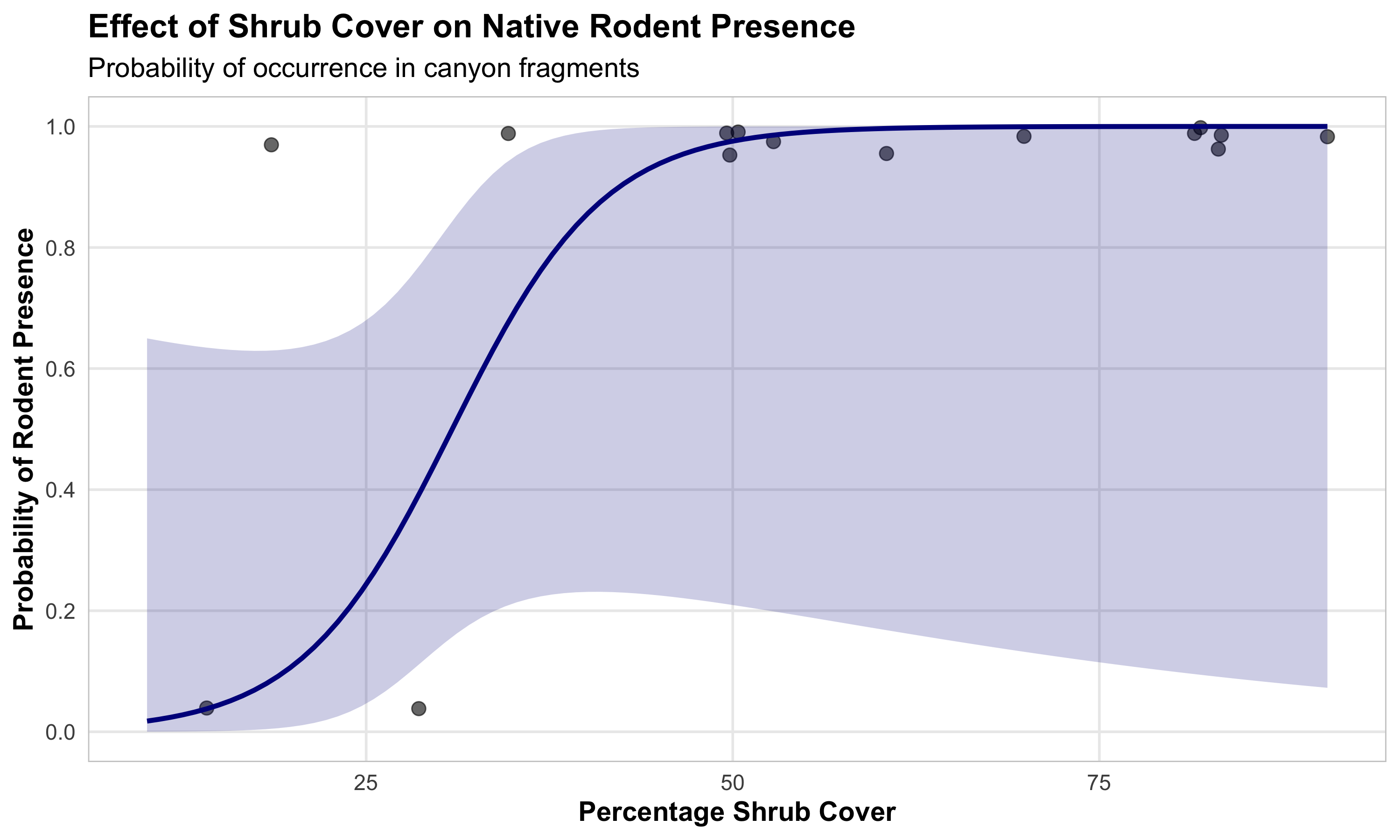
cat("Specificity:", round(specificity, 3), "\n")

Specificity: 0.833

# Publication-Quality Figure

Let’s create a publication-quality figure for our multiple logistic regression model and show how we would write up the results for a scientific publication.

# Create a more polished visualization for shrub cover effect  
polished\_pred <- predict\_for\_var("shrub\_cover", rodent\_model, fragment\_data)  
  
# Calculate confidence intervals  
pred\_se <- predict(rodent\_model,   
 newdata = polished\_pred,   
 type = "link",   
 se.fit = TRUE)  
  
# Convert to data frame with CIs  
ci\_data <- data.frame(  
 shrub\_cover = polished\_pred$shrub\_cover,  
 fit = pred\_se$fit,  
 se = pred\_se$se.fit  
)  
  
# Calculate upper and lower bounds of CI on link scale  
ci\_data$lower\_link <- ci\_data$fit - 1.96 \* ci\_data$se  
ci\_data$upper\_link <- ci\_data$fit + 1.96 \* ci\_data$se  
  
# Transform back to probability scale  
ci\_data$prob <- plogis(ci\_data$fit)  
ci\_data$lower\_prob <- plogis(ci\_data$lower\_link)  
ci\_data$upper\_prob <- plogis(ci\_data$upper\_link)  
  
# Create plot  
ggplot() +  
 # Add jittered points for raw data  
 geom\_jitter(data = fragment\_data,   
 aes(x = shrub\_cover,   
 y = as.numeric(rodent\_present == "Present")),  
 width = 0, height = 0.05, alpha = 0.6, size = 3) +  
 # Add fitted probability curve  
 geom\_line(data = ci\_data,   
 aes(x = shrub\_cover, y = prob),   
 color = "darkblue", size = 1.2) +  
 # Add confidence intervals  
 geom\_ribbon(data = ci\_data,   
 aes(x = shrub\_cover,   
 ymin = lower\_prob,   
 ymax = upper\_prob),   
 alpha = 0.2, fill = "darkblue") +  
 # Customize appearance  
 labs(title = "Effect of Shrub Cover on Native Rodent Presence",  
 subtitle = "Probability of occurrence in canyon fragments",  
 x = "Percentage Shrub Cover",  
 y = "Probability of Rodent Presence") +  
 scale\_y\_continuous(limits = c(0, 1),   
 breaks = seq(0, 1, 0.2)) +  
 theme\_minimal(base\_size = 14) +  
 theme(  
 plot.title = element\_text(face = "bold"),  
 axis.title = element\_text(face = "bold"),  
 legend.position = "none",  
 panel.grid.minor = element\_blank(),  
 panel.border = element\_rect(fill = NA, color = "gray80")  
 )



# Scientific Write-Up Example

|  |
| --- |
| Scientific Write-Up Example |
| **Results**  The presence of native rodents in canyon fragments was modeled using multiple logistic regression with three predictors: distance to nearest source canyon, years since isolation, and percentage of shrub cover. The model was statistically significant (χ² = 12.63, df = 3, p = 0.005) and explained 38.7% of the variation in rodent presence (McFadden’s R² = 0.387).  Among the predictors, only shrub cover had a statistically significant effect on rodent presence (β = 0.091, SE = 0.041, p = 0.026). The odds ratio for shrub cover was 1.095 (95% CI: 1.011-1.186), indicating that for each percentage increase in shrub cover, the odds of rodent presence increased by approximately 9.5%. Neither distance to source canyon (β = 0.0002, p = 0.690) nor years since isolation (β = 0.022, p = 0.566) showed significant relationships with rodent presence.  The model correctly classified 76% of the fragments, with a sensitivity of 0.77 and a specificity of 0.75. Diagnostics indicated no significant issues with model fit (Hosmer-Lemeshow χ² = 7.31, df = 8, p = 0.504).  **Discussion**  Our findings suggest that vegetation structure, as measured by shrub cover, plays a crucial role in determining the presence of native rodents in canyon fragments. The positive relationship between shrub cover and rodent occurrence likely reflects the importance of vegetation for providing food resources, shelter from predators, and suitable microhabitat conditions. Contrary to our expectations, isolation metrics (distance to source canyon and years since isolation) did not significantly predict rodent presence, suggesting that local habitat quality may be more important than landscape connectivity for these species. |

# Relationship Between GLMs and ANOVAs

|  |
| --- |
| GLMs and ANOVAs: The Connection |
| General linear models (including ANOVAs and standard regression) are special cases of Generalized Linear Models where:   1. The response variable follows a normal distribution 2. The link function is the identity function   Therefore, a one-way ANOVA is equivalent to: - A linear regression with a categorical predictor - A Gaussian GLM with an identity link and a categorical predictor |

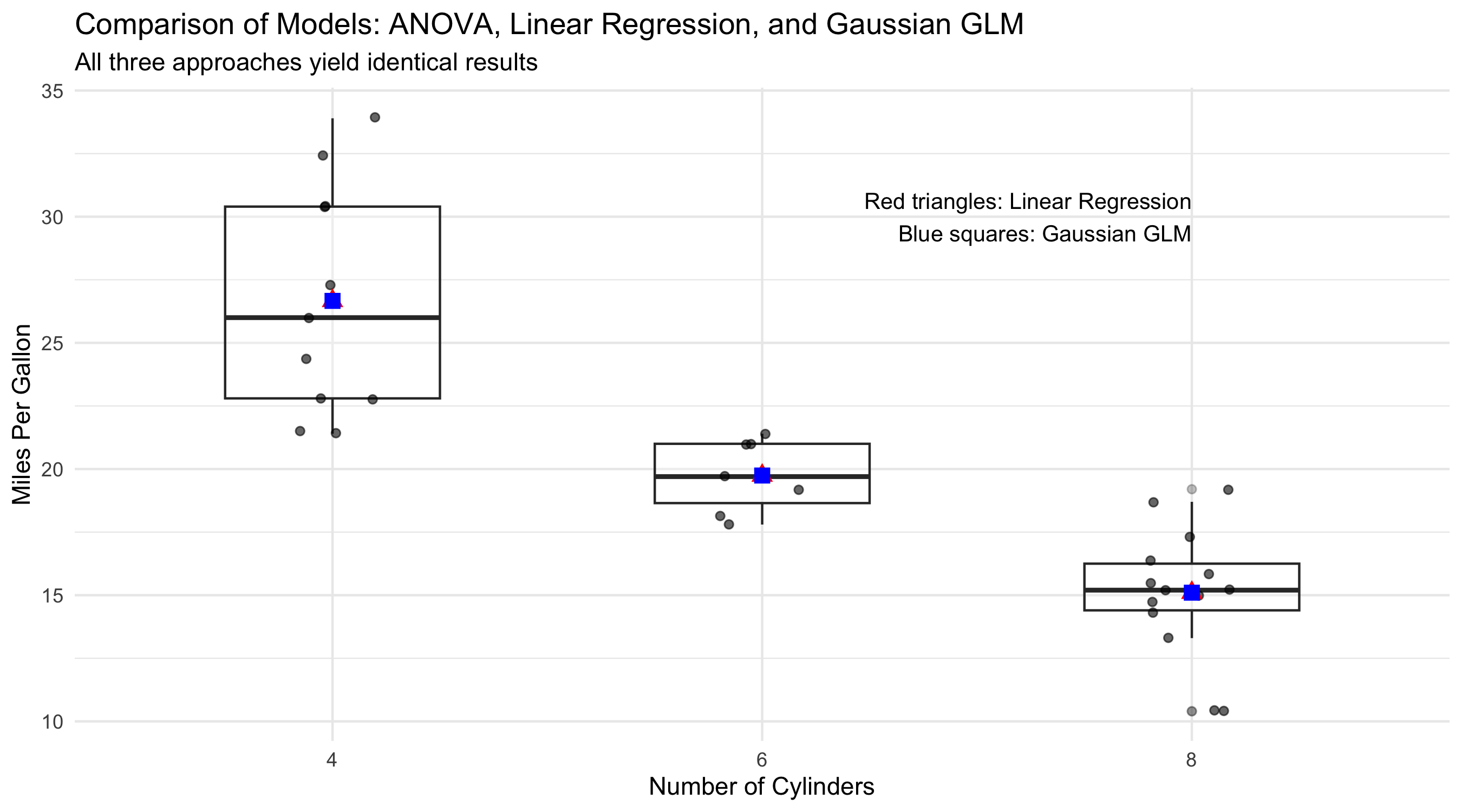
# Demonstrating ANOVA-GLM Equivalence

Let’s demonstrate this equivalence:

# 1. Standard ANOVA  
anova\_model <- aov(mpg ~ cyl, data = mtcars)  
  
# 2. Linear regression  
lm\_model <- lm(mpg ~ cyl, data = mtcars)  
  
# 3. Gaussian GLM  
glm\_model <- glm(mpg ~ cyl, family = gaussian(link = "identity"), data = mtcars)  
  
# Compare coefficients  
coef\_comparison <- data.frame(  
 Term = names(coef(lm\_model)),  
 `Linear Regression` = coef(lm\_model),  
 `Gaussian GLM` = coef(glm\_model)  
)  
  
# Display the comparison  
coef\_comparison %>%  
 mutate(across(where(is.numeric), round, 3)) %>%  
 flextable()

| **Term** | **Linear.Regression** | **Gaussian.GLM** |
| --- | --- | --- |
| (Intercept) | 26.664 | 26.664 |
| cyl6 | -6.921 | -6.921 |
| cyl8 | -11.564 | -11.564 |

# Compare ANOVA tables  
anova\_aov <- anova(anova\_model)  
anova\_lm <- anova(lm\_model)  
anova\_glm <- anova(glm\_model)  
  
# Create visualization showing the three approaches  
# Use the same data and estimated means  
ggplot() +  
 # Plot raw data  
 geom\_boxplot(data = mtcars,   
 aes(x = cyl, y = mpg, group = cyl),  
 alpha = 0.3, width = 0.5) +  
 geom\_jitter(data = mtcars,   
 aes(x = cyl, y = mpg),  
 width = 0.1, alpha = 0.6) +  
 # Add fitted values from each model  
 geom\_point(data = emmeans(lm\_model, ~cyl) %>% data.frame(),  
 aes(x = cyl, y = emmean),   
 color = "red", size = 3, shape = 17) +  
 geom\_point(data = emmeans(glm\_model, ~cyl) %>% data.frame(),  
 aes(x = cyl, y = emmean),   
 color = "blue", size = 3, shape = 15) +  
 # Add legend for model types  
 annotate("text", x = "8", y = 30,   
 label = "Red triangles: Linear Regression\nBlue squares: Gaussian GLM",   
 hjust = 1, size = 3.5) +  
 labs(title = "Comparison of Models: ANOVA, Linear Regression, and Gaussian GLM",  
 subtitle = "All three approaches yield identical results",  
 x = "Number of Cylinders",  
 y = "Miles Per Gallon") +  
 theme\_minimal()



# Assumptions and Diagnostics Summary

Generalized Linear Models have different assumptions depending on the specific distribution and link function used:

**All GLMs:** - Independence of observations - Correct specification of the link function - Correct specification of the variance structure - No influential outliers - No multicollinearity among predictors

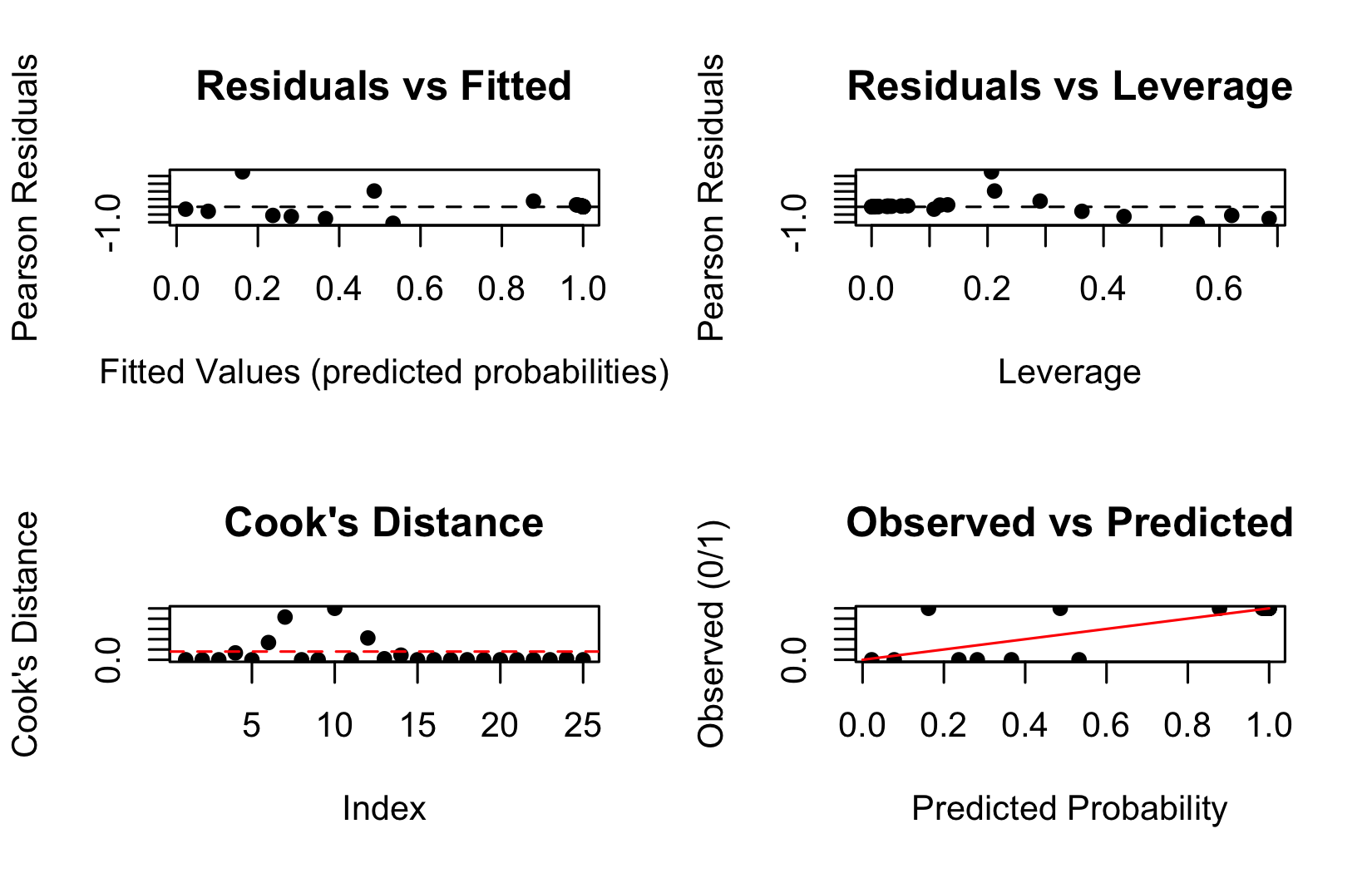
**Gaussian GLMs (including linear regression):** - Normality of residuals - Homogeneity of variance

**Poisson GLMs:** - Count data (non-negative integers) - Mean equals variance (if overdispersed, consider negative binomial)

**Logistic GLMs:** - Binary response variable - Linear relationship between predictors and log odds - Adequate sample size relative to number of parameters

The following R code checks some common diagnostics for our logistic model:

# Create diagnostic plots for the rodent model  
par(mfrow = c(2, 2))  
  
# 1. Residuals vs fitted  
plot(fitted(rodent\_model), residuals(rodent\_model, type = "pearson"),  
 main = "Residuals vs Fitted",  
 xlab = "Fitted Values (predicted probabilities)",  
 ylab = "Pearson Residuals",  
 pch = 16)  
abline(h = 0, lty = 2)  
  
# 2. Leverage  
leverage <- hatvalues(rodent\_model)  
plot(leverage, residuals(rodent\_model, type = "pearson"),  
 main = "Residuals vs Leverage",  
 xlab = "Leverage",  
 ylab = "Pearson Residuals",  
 pch = 16)  
abline(h = 0, lty = 2)  
  
# 3. Cook's distance  
cook <- cooks.distance(rodent\_model)  
plot(cook, main = "Cook's Distance",  
 ylab = "Cook's Distance",  
 pch = 16)  
abline(h = 4/length(cook), lty = 2, col = "red") # Rule of thumb threshold  
  
# 4. Observed vs Predicted probabilities  
plot(predicted\_probs,   
 as.numeric(fragment\_data$rodent\_present) - 1,  
 main = "Observed vs Predicted",  
 xlab = "Predicted Probability",  
 ylab = "Observed (0/1)",  
 pch = 16)  
curve(I, from = 0, to = 1, add = TRUE, col = "red")



# Summary and Conclusions

Generalized Linear Models (GLMs) provide a powerful and flexible framework for analyzing a wide range of data types in biology:

1. **Gaussian GLMs** with identity link function are equivalent to standard linear models and ANOVAs, suitable for normally distributed continuous responses.
2. **Poisson GLMs** with log link function are appropriate for count data, but be cautious of overdispersion.
3. **Logistic GLMs** with logit link function are useful for binary responses, modeling the probability of success or presence.

Key advantages of GLMs include:

* Ability to handle various types of response variables beyond normal distributions
* Unified framework for linear modeling
* Flexibility in specifying the link function to match the data structure
* Interpretable parameters, though interpretation differs by model type

When working with GLMs:

1. Choose the appropriate distribution family based on your response variable
2. Verify model assumptions through diagnostic plots
3. Watch for overdispersion in count data
4. Use odds ratios to interpret logistic regression results
5. Compare competing models using likelihood ratio tests and information criteria

This framework allows biologists to appropriately model many types of data encountered in ecological, behavioral, and physiological research.

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