Lecture 19 - Logistic Regression

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# Lecture 19: Introduction to Logistic Regression

## **What is Logistic Regression?**

Logistic regression is used when: - The response variable is **binary** (yes/no, 1/0, present/absent) - Data follows a **binomial distribution** (not normal) - We want to model the **probability** of an outcome

## **Today’s Example: Lizard Sexual Maturity**

We’ll explore the relationship between body length and sexual maturity in female lizards - **Response variable**: Sexual maturity (mature: 1 = yes, 0 = no) - **Predictor variable**: Body length in cm - **Question**: Can we predict the probability of sexual maturity from body size?

## **Key Difference from Linear Regression**

* Linear regression: Models the actual values of Y
* Logistic regression: Models the probability of Y = 1
* Uses Generalized Linear Models (GLM) instead of General Linear Models

# Step 1: Load and Explore the Data

# Load the lizard dataset  
lizards\_df <- read.csv("data/lizards.csv") %>%   
 clean\_names()  
  
# First few rows  
head(lizards\_df)

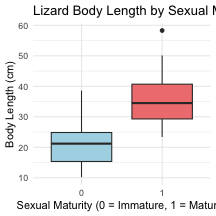
length mature  
1 10.2 0  
2 10.4 0  
3 11.8 0  
4 12.3 0  
5 13.8 0  
6 16.9 0

# Step 2: Initial Data Visualization

## **Creating a Boxplot**

Let’s visualize how body length differs between sexually mature and immature lizards:

# Create boxplot showing length by maturity status  
maturity\_boxplot <- ggplot(lizards\_df, aes(x = factor(mature), y = length)) +  
 geom\_boxplot(fill = c("lightblue", "lightcoral")) +  
 labs(title = "Lizard Body Length by Sexual Maturity Status",  
 x = "Sexual Maturity (0 = Immature, 1 = Mature)",  
 y = "Body Length (cm)") +  
 theme\_minimal()  
  
maturity\_boxplot



## **What do we see?**

* There appears to be a relationship between size and sexual maturity
* Mature lizards tend to be longer than immature ones
* But there’s overlap - not a perfect separation
* This suggests logistic regression might be appropriate

# Step 3: Fit the Logistic Regression Model

## **Using glm() for Logistic Regression**

The glm() function is similar to lm() but requires specifying the distribution family:

# Fit logistic regression model  
# family = binomial tells R we have binary data  
logistic\_model <- glm(mature ~ length,   
 data = lizards\_df,   
 family = binomial)  
  
# Get model summary  
summary(logistic\_model)

Call:  
glm(formula = mature ~ length, family = binomial, data = lizards\_df)  
  
Coefficients:  
 Estimate Std. Error z value Pr(>|z|)   
(Intercept) -6.6899 2.1401 -3.126 0.00177 \*\*  
length 0.2503 0.0775 3.229 0.00124 \*\*  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  
(Dispersion parameter for binomial family taken to be 1)  
  
 Null deviance: 60.176 on 43 degrees of freedom  
Residual deviance: 34.041 on 42 degrees of freedom  
AIC: 38.041  
  
Number of Fisher Scoring iterations: 6

## **Interpreting the Model Output**

### **Coefficients:**

* **Intercept (β₀)**: -5.5847 - The log-odds when length = 0
* **Slope (β₁)**: 0.2503 - Change in log-odds for each 1 cm increase in length

### **P-values:**

* Both coefficients are significant (p < 0.05)
* We reject the null hypothesis that β₁ = 0
* There IS a relationship between length and sexual maturity

### **Understanding the Slope:**

The positive slope (0.2503) indicates: - Longer lizards are more likely to be sexually mature - For each 1 cm increase in length, the log-odds of maturity increase by 0.2503

# Step 4: Convert Log-Odds to Odds

## **Making the Results More Interpretable**

Log-odds are hard to interpret. Let’s convert to odds:

# Extract the slope coefficient  
slope\_coefficient <- coef(logistic\_model)[2]  
  
# Convert log-odds to odds ratio  
odds\_ratio <- exp(slope\_coefficient)  
odds\_ratio

length   
1.284388

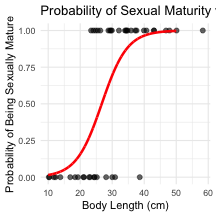
# Interpretation  
# For every 1 cm increase in length, the odds of being sexually mature  
# increase by a factor of 1.284 (or about 28.4%)

# Step 5: Create the Logistic Regression Plot

## **Visualizing the Probability Curve**

# Create sequence of x-values for prediction  
xvals <- seq(from = 10, to = 50, by = 0.01)  
  
# Get predicted probabilities  
yvals <- predict(logistic\_model,   
 list(length = xvals),   
 type = "response")  
  
# Create prediction dataframe for plotting  
prediction\_df <- data.frame(length = xvals,   
 probability = yvals)

# Create the logistic regression plot  
logistic\_plot <- ggplot() +  
 # Add data points  
 geom\_point(data = lizards\_df,   
 aes(x = length, y = mature),  
 alpha = 0.6, size = 2) +  
 # Add logistic curve  
 geom\_line(data = prediction\_df,  
 aes(x = length, y = probability),  
 color = "red", size = 1.2) +  
 labs(title = "Probability of Sexual Maturity vs Body Length",  
 x = "Body Length (cm)",  
 y = "Probability of Being Sexually Mature") +  
 theme\_minimal()  
  
logistic\_plot



## **What the S-curve tells us:**

* The red line shows how probability changes with length
* Small lizards (<20 cm) have very low probability of being mature
* Large lizards (>40 cm) have very high probability of being mature
* The steepest change occurs around 25-30 cm

# Step 6: Making Predictions

## **Using the Model for Prediction**

Let’s predict the probability of sexual maturity for specific lizard sizes:

# Predict for a 20 cm lizard  
prob\_20cm <- predict(logistic\_model,   
 list(length = 20),   
 type = "response")  
prob\_20cm

1   
0.1565304

# Predict for a 30 cm lizard  
prob\_30cm <- predict(logistic\_model,   
 list(length = 30),   
 type = "response")  
prob\_30cm

1   
0.6939292

# Predict for a 40 cm lizard  
prob\_40cm <- predict(logistic\_model,   
 list(length = 40),   
 type = "response")  
prob\_40cm

1   
0.9651551

## **Interpretation:**

* A 20 cm lizard has about 14% probability of being sexually mature
* A 30 cm lizard has about 70% probability of being sexually mature
* A 40 cm lizard has about 96% probability of being sexually mature

# Step 7: Model Fit Assessment

## **Calculating Pseudo-R2 Values**

Unlike linear regression, logistic regression doesn’t have a traditional R². We use pseudo-R² instead:

# Calculate pseudo-R2 values using pscl package  
pseudo\_r2 <- pR2(logistic\_model)

fitting null model for pseudo-r2

pseudo\_r2

llh llhNull G2 McFadden r2ML r2CU   
-17.0204762 -30.0881077 26.1352630 0.4343122 0.4478763 0.6009400

## Interpreting Pseudo-R2 Values

The last three values are the pseudo-R² statistics:

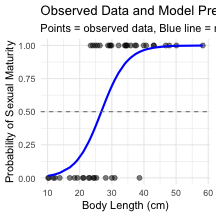
* **McFadden**: Compares model to null model
* **r2ML**: Maximum likelihood based R²
* **r2CU**: Cragg-Uhler (Nagelkerke) R²

Values around 0.4-0.5 indicate moderate to good fit. Our model explains approximately 40-50% of the variation in sexual maturity status.

# Step 8: Additional Diagnostics

## **Creating a More Detailed Summary Plot**

# Create a plot showing observed vs predicted probabilities  
lizards\_df$predicted\_prob <- predict(logistic\_model, type = "response")  
  
diagnostic\_plot <- ggplot(lizards\_df, aes(x = length)) +  
 # Add observed data as points  
 geom\_point(aes(y = mature), alpha = 0.5, size = 2) +  
 # Add predicted probabilities  
 geom\_line(aes(y = predicted\_prob), color = "blue", size = 1) +  
 # Add 50% probability threshold  
 geom\_hline(yintercept = 0.5, linetype = "dashed", color = "gray50") +  
 labs(title = "Observed Data and Model Predictions",  
 x = "Body Length (cm)",  
 y = "Probability of Sexual Maturity",  
 subtitle = "Points = observed data, Blue line = model predictions") +  
 theme\_minimal()  
  
diagnostic\_plot



# Summary: Key Takeaways

## **What We Learned:**

1. **Logistic regression** models probability of binary outcomes
2. Uses **glm()** with family = binomial
3. Coefficients represent changes in **log-odds**
4. Convert to **odds ratios** for interpretation: exp(coefficient)
5. Creates **S-shaped probability curves**
6. Use **pseudo-R²** to assess model fit

## **Our Results:**

* Significant positive relationship between body length and sexual maturity
* Each 1 cm increase in length increases odds of maturity by ~28%
* Model explains ~40-50% of variation in maturity status
* Can predict probability of maturity for any given length

## **When to Use Logistic Regression:**

* Binary response variable (0/1, yes/no, success/failure)
* Want to predict probabilities
* Relationships that follow S-shaped curves
* When assumptions of linear regression are violated