Lecture 19 - Logistic Regression

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Lecture 19: Introduction to Logistic Regression

What is Logistic Regression?

Logistic regression is used when: - The response variable is **binary** (yes/no, 1/0, present/absent) - Data follows a **binomial distribution** (not normal) - We want to model the **probability** of an outcome

Today's Example: Lizard Sexual Maturity

We'll explore the relationship between body length and sexual maturity in female lizards - **Response variable**: Sexual maturity (mature: 1 = yes, 0 = no) - **Predictor variable**: Body length in cm - **Question**: Can we predict the probability of sexual maturity from body size?

Key Difference from Linear Regression

- Linear regression: Models the actual values of Y
- Logistic regression: Models the probability of Y = 1
- Uses Generalized Linear Models (GLM) instead of General Linear Models

Step 1: Load and Explore the Data

```
# Load the lizard dataset
lizards_df <- read.csv("data/lizards.csv") %>%
  clean_names()

# First few rows
head(lizards_df)
```

```
  length mature
1   10.2      0
2   10.4      0
3   11.8      0
4   12.3      0
5   13.8      0
6   16.9      0
```

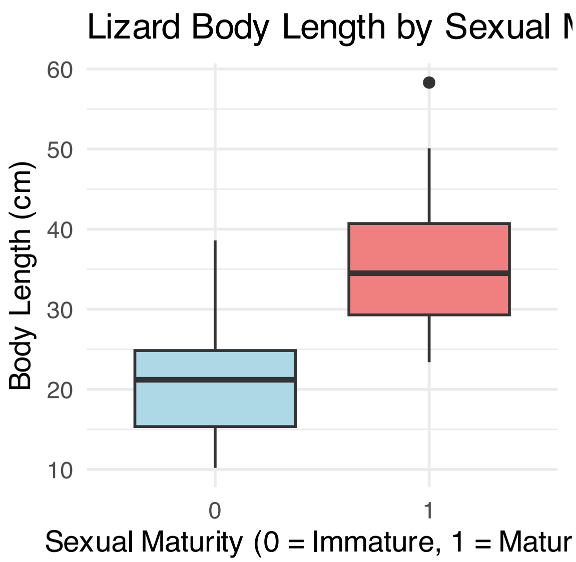
Step 2: Initial Data Visualization

Creating a Boxplot

Let's visualize how body length differs between sexually mature and immature lizards:

```
# Create boxplot showing length by maturity status
maturity_boxplot <- ggplot(lizards_df, aes(x = factor(mature), y = length)) +
  geom_boxplot(fill = c("lightblue", "lightcoral")) +
  labs(title = "Lizard Body Length by Sexual Maturity Status",
       x = "Sexual Maturity (0 = Immature, 1 = Mature)",
       y = "Body Length (cm)") +
```

```
  theme_minimal()

maturity_boxplot
```



What do we see?

- There appears to be a relationship between size and sexual maturity
- Mature lizards tend to be longer than immature ones
- But there's overlap - not a perfect separation
- This suggests logistic regression might be appropriate

Step 3: Fit the Logistic Regression Model

Using glm() for Logistic Regression

The `glm()` function is similar to `lm()` but requires specifying the distribution family:

```
# Fit logistic regression model
# family = binomial tells R we have binary data
logistic_model <- glm(mature ~ length,
                      data = lizards_df,
                      family = binomial)

# Get model summary
summary(logistic_model)
```

```

Call:
glm(formula = mature ~ length, family = binomial, data = lizards_df)

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  -6.6899     2.1401  -3.126  0.00177 **
length        0.2503     0.0775   3.229  0.00124 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 60.176  on 43  degrees of freedom
Residual deviance: 34.041  on 42  degrees of freedom
AIC: 38.041

Number of Fisher Scoring iterations: 6
```

Interpreting the Model Output

Coefficients:

- **Intercept (β_0)**: −5.5847 - The log-odds when length = 0
- **Slope (β_1)**: 0.2503 - Change in log-odds for each 1 cm increase in length

P-values:

- Both coefficients are significant ($p < 0.05$)
- We reject the null hypothesis that $\beta_1 = 0$
- There IS a relationship between length and sexual maturity

Understanding the Slope:

The positive slope (0.2503) indicates: - Longer lizards are more likely to be sexually mature - For each 1 cm increase in length, the log-odds of maturity increase by 0.2503

Step 4: Convert Log-Odds to Odds

Making the Results More Interpretable

Log-odds are hard to interpret. Let's convert to odds:

```
# Extract the slope coefficient
slope_coefficient <- coef(logistic_model)[2]

# Convert log-odds to odds ratio
odds_ratio <- exp(slope_coefficient)
odds_ratio
```

```
  length
1.284388
```

```
# Interpretation
# For every 1 cm increase in length, the odds of being sexually mature
# increase by a factor of 1.284 (or about 28.4%)
```

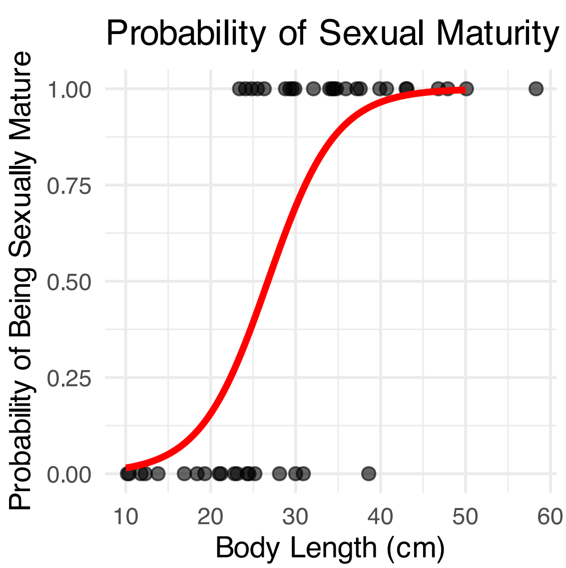
Step 5: Create the Logistic Regression Plot

Visualizing the Probability Curve

```
# Create sequence of x-values for prediction
xvals <- seq(from = 10, to = 50, by = 0.01)

# Get predicted probabilities
yvals <- predict(logistic_model,
                 list(length = xvals),
```

```
                  type = "response")

# Create prediction dataframe for plotting
prediction_df <- data.frame(length = xvals,
                           probability = yvals)
```

```
# Create the logistic regression plot
logistic_plot <- ggplot() +
  # Add data points
  geom_point(data = lizards_df,
             aes(x = length, y = mature),
             alpha = 0.6, size = 2) +
  # Add logistic curve
  geom_line(data = prediction_df,
            aes(x = length, y = probability),
            color = "red", size = 1.2) +
  labs(title = "Probability of Sexual Maturity vs Body Length",
       x = "Body Length (cm)",
       y = "Probability of Being Sexually Mature") +
  theme_minimal()

logistic_plot
```



What the S-curve tells us:

- The red line shows how probability changes with length
- Small lizards (<20 cm) have very low probability of being mature
- Large lizards (>40 cm) have very high probability of being mature
- The steepest change occurs around 25-30 cm

Step 6: Making Predictions

Using the Model for Prediction

Let's predict the probability of sexual maturity for specific lizard sizes:

```r
# Predict for a 20 cm lizard
prob_20cm <- predict(logistic_model,
                     list(length = 20),
                     type = "response")
prob_20cm
```

```
        1
0.1565304
```

```r
# Predict for a 30 cm lizard
prob_30cm <- predict(logistic_model,
                     list(length = 30),
                     type = "response")
prob_30cm
```

```
        1
0.6939292
```

```r
# Predict for a 40 cm lizard
prob_40cm <- predict(logistic_model,
                     list(length = 40),
                     type = "response")
prob_40cm
```

```
        1
0.9651551
```

Interpretation:

- A 20 cm lizard has about 14% probability of being sexually mature
- A 30 cm lizard has about 70% probability of being sexually mature
- A 40 cm lizard has about 96% probability of being sexually mature

Step 7: Model Fit Assessment

Calculating Pseudo-R2 Values

Unlike linear regression, logistic regression doesn't have a traditional R^2. We use pseudo-R^2 instead:

```r
# Calculate pseudo-R2 values using pscl package
pseudo_r2 <- pR2(logistic_model)
```

```
fitting null model for pseudo-r2
```

```r
pseudo_r2
```

```
        llh     llhNull          G2    McFadden        r2ML        r2CU
-17.0204762 -30.0881077  26.1352630   0.4343122   0.4478763   0.6009400
```

Interpreting Pseudo-R2 Values

The last three values are the pseudo-R^2 statistics:

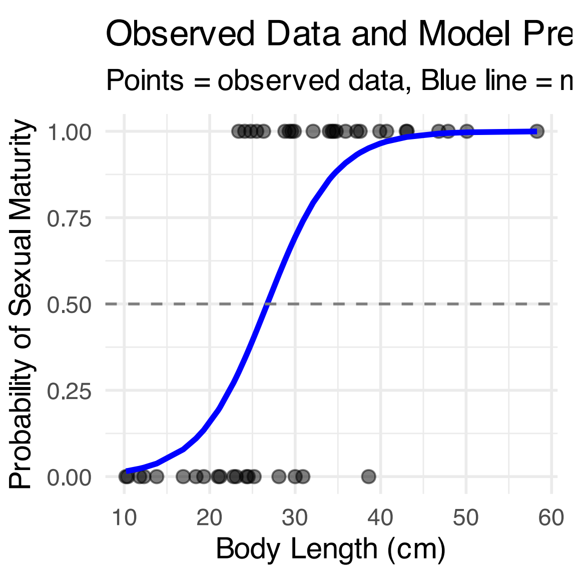
- **McFadden**: Compares model to null model
- **r2ML**: Maximum likelihood based R^2
- **r2CU**: Cragg-Uhler (Nagelkerke) R^2

Values around 0.4-0.5 indicate moderate to good fit. Our model explains approximately 40-50% of the variation in sexual maturity status.

Step 8: Additional Diagnostics

Creating a More Detailed Summary Plot

```r
# Create a plot showing observed vs predicted probabilities
lizards_df$predicted_prob <- predict(logistic_model, type = "response")

diagnostic_plot <- ggplot(lizards_df, aes(x = length)) +
  # Add observed data as points
  geom_point(aes(y = mature), alpha = 0.5, size = 2) +
  # Add predicted probabilities
  geom_line(aes(y = predicted_prob), color = "blue", size = 1) +
  # Add 50% probability threshold
  geom_hline(yintercept = 0.5, linetype = "dashed", color = "gray50") +
  labs(title = "Observed Data and Model Predictions",
       x = "Body Length (cm)",
       y = "Probability of Sexual Maturity",
       subtitle = "Points = observed data, Blue line = model predictions") +
  theme_minimal()

diagnostic_plot
```



Summary: Key Takeaways

What We Learned:

1. **Logistic regression** models probability of binary outcomes
2. Uses **glm()** with `family = binomial`
3. Coefficients represent changes in **log-odds**

4. Convert to **odds ratios** for interpretation: exp(coefficient)
5. Creates **S-shaped probability curves**
6. Use **pseudo-R^2** to assess model fit

Our Results:

- Significant positive relationship between body length and sexual maturity
- Each 1 cm increase in length increases odds of maturity by ~28%
- Model explains ~40-50% of variation in maturity status
- Can predict probability of maturity for any given length

When to Use Logistic Regression:

- Binary response variable (0/1, yes/no, success/failure)
- Want to predict probabilities
- Relationships that follow S-shaped curves
- When assumptions of linear regression are violated